A Novel and Simple Discrete Symmetry for Non-zero $\theta_{13}$

Yang-Hwan, Ahn (KIAS)

Collaboration with Seungwon Baek and Paolo Gondolo

NRF workshop
Yonsei Univ., Jun 7-8, 2012
We propose a simple model based on A4 flavor symmetry in a type-I seesaw mechanism.
Where Do we Stand?

Latest 3 neutrino global analysis including atm., solar, reactor (Double Chooz), LBL (T2K/MINOS)

And the latest Daya Bay and RENO results


The hypothesis $\theta_{13} = 0$ is now rejected at a 8$\sigma$ significance level.

<table>
<thead>
<tr>
<th>parameter</th>
<th>best fit ±1$\sigma$</th>
<th>2$\sigma$</th>
<th>3$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_{21}^2$ [$10^{-5}$eV$^2$]</td>
<td>7.62 ± 0.19</td>
<td>7.27–8.01</td>
<td>7.12–8.20</td>
</tr>
<tr>
<td>$\Delta m_{31}^2$ [$10^{-3}$eV$^2$]</td>
<td>2.53$^{+0.08}_{-0.10}$</td>
<td>2.34 – 2.69</td>
<td>2.26 – 2.77</td>
</tr>
<tr>
<td>$\Delta m_{31}^2$ $^{(2.40+0.10}_{-0.07}$</td>
<td>$(2.25 – 2.59)$</td>
<td>$(2.15 – 2.68)$</td>
<td></td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>0.320$^{+0.015}_{-0.017}$</td>
<td>0.29–0.35</td>
<td>0.27–0.37</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>0.49$^{+0.08}_{-0.05}$</td>
<td>0.41–0.62</td>
<td>0.39–0.64</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>0.53$^{+0.05}_{-0.07}$</td>
<td>0.42–0.62</td>
<td></td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>0.026$^{+0.003}_{-0.004}$</td>
<td>0.019–0.033</td>
<td>0.015–0.036</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>0.027$^{+0.003}_{-0.004}$</td>
<td>0.020–0.034</td>
<td>0.016–0.037</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$(0.83^{+0.54}_{-0.64}) \pi$</td>
<td>$0 – 2\pi$</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>$0.07\pi \alpha$</td>
<td>$0 – 2\pi$</td>
<td></td>
</tr>
</tbody>
</table>
Where Do we Stand?

Latest 3 neutrino global analysis including atm., solar, reactor (Double Chooz), LBL (T2K/MINOS)

And the latest Daya Bay and RENO results


The hypothesis $\theta_{13}=0$ is now rejected at a 8$\sigma$ significance level.

<table>
<thead>
<tr>
<th>parameter</th>
<th>best fit $\pm 1\sigma$</th>
<th>2$\sigma$</th>
<th>3$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_{21}^2$ [$10^{-5}\text{eV}^2$]</td>
<td>$7.62 \pm 0.19$</td>
<td>7.27–8.01</td>
<td>7.12–8.20</td>
</tr>
<tr>
<td>$\Delta m_{31}^2$ [$10^{-3}\text{eV}^2$]</td>
<td>$2.53^{+0.08}_{-0.10}$</td>
<td>2.34–2.69</td>
<td>2.26–2.77</td>
</tr>
<tr>
<td>$- (2.40^{+0.10}_{-0.07})$</td>
<td>$- (2.25–2.59)$</td>
<td>$- (2.15–2.68)$</td>
<td></td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>$0.320^{+0.015}_{-0.017}$</td>
<td>0.29–0.35</td>
<td>0.27–0.37</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>$0.49^{+0.05}_{-0.05}$</td>
<td>0.41–0.62</td>
<td>0.39–0.64</td>
</tr>
<tr>
<td>$0.53^{+0.05}_{-0.07}$</td>
<td>0.42–0.62</td>
<td>0.39–0.64</td>
<td></td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>$0.026^{+0.003}_{-0.004}$</td>
<td>0.019–0.033</td>
<td>0.015–0.036</td>
</tr>
<tr>
<td>$0.027^{+0.003}_{-0.004}$</td>
<td>0.020–0.034</td>
<td>0.016–0.037</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>$(0.83^{+0.54}_{-0.64}) \pi$</td>
<td>$0 - 2\pi$</td>
<td>$0 - 2\pi$</td>
</tr>
</tbody>
</table>

Relatively large 7°-11°
All data can be explained in terms of oscillation between just 3 known species

Their neutrino species (\(\nu_e\), \(\nu_\mu\), \(\nu_\tau\)) have definite masses:

\[\nu_{\text{flavor}} = U \nu_m, \quad |\Delta m^2_{31}| \gg |\Delta m^2_{21}|\]

Two possible orderings of neutrino masses

\[\Delta m^2_{21}\]

Earth matter effects (LBL)

\[\Delta m^2_{31}\]

Quasi-Degenerate case

And one lepton flavor can convert to another

\[-\mathcal{L}_{\text{cc}} = \frac{g}{\sqrt{2}} \left( e \quad \mu \quad \tau \right)_L \gamma^\mu \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^-_\mu + \text{h.c.}\]
Pontecorvo–Maki–NaKagawa–Sakata (PMNS) Matrix

\[
U_{\text{PMNS}} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta_{CP}} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

Atmospheric and LBL accelerator \quad SBL reactor \quad Solar and LBL reactor

Majorana phases \quad \longleftrightarrow \quad Neutrinoless Double beta decay

Not observable in Neutrino oscillation

A probe of lepton number violation in low energy

2012-06-09
Seesaw

A simple and attractive explanation of the smallness of $\nu$ mass:

SEESAW MECHANISM:

(i) SM+RH $\nu$ (EW singlet)  
(Minkowski 1977)

(ii) SM+SU(2) Triplet Higgs  
(Konetschny, Kummer 1977)

(iii) SM+SU(2) Triplet fermions  

while $\tau$ of the EW int. keeps invariant SU(2) $\times$ U(1)

$3 \times 3$ Seesaw model has 18 parameters: 12 real+6 phases  
(cf. Casas Ibarra $Y_D = U_{PMNS}^* \sqrt{m_\nu^d R} \sqrt{M_R^d} / \nu$)

Integrating out the heavy fermions leaves us with observable mass matrix  
(9 observables: 6 real+3 phases)

Half of the parameters of the model get lost at low-\(E\)

The 3 low-\(E\) CP-violating phases depend, in general, on all 6 seesaw phases.
Seesaw

◆ A simple and attractive explanation of the smallness of $\nu$ mass:

SEESAW MECHANISM:

(i) SM+RH $\nu$ (EW singlet) 
(Minkowski 1977)
(ii) SM+SU(2) Triplet Higgs
(Konetschnv, Kummer 1977)
(iii) SM+SU(2) Triplet fermions

while Ł of the EW int. keeps invariant $SU(2) \times U(1)$

$3 \times 3$ Seesaw model has 18 parameters: 12 real+6 phases

\[ Y_D = U_{PMNS}^* \sqrt{m_\nu^d} R \sqrt{M_R^d} / \Delta \]

Integrating out the heavy fermions leaves us with observable mass matrix
(9 observables: 6 real+3 phases)

- Half of the parameters of the model get lost at low-$E$
- The 3 low-$E$ CP-violating phases depend, in general, on all 6 seesaw phases.

◆ Seesaw has another appealing feature, built-in mechanism for generating BAU so-called leptogenesis

The effects of high-$E$ CP-violating phases control the generation of the BAU
in the leptogenesis scenario, in $|<m>|$ and in the leptonic CP-violating rephasing invariant $J_{cp}$.
Seesaw

A simple and attractive explanation of the smallness of $\nu$ mass:

SEESAW MECHANISM:
(i) SM+RH $\nu$ (EW singlet) (Minkowski 1977)
(ii) SM+SU(2) Triplet Higgs (Konetschnv,Kummer 1977)

while $\tau$ of the EW int. keeps invariant SU(2)$\times$U(1)

$3\times 3$ Seesaw model has 18 parameters: 12 real+6 phases

(cf. Casas Ibarra $Y_D = U_{PMNS}^* \sqrt{m_\nu} R \sqrt{M_R^d} / \sqrt{\nu}$)

Integrating out the heavy fermions leaves us with observable mass matrix
(9 observables: 6 real+3 phases)
Half of the parameters of the model get lost at low-$E$
The $3$ low-$E$ CP-violating phases depend, in general, on all $6$ seesaw phases.

Seesaw has another appealing feature, built-in mechanism for generating BAU
so-called leptogenesis

The effects of high-$E$ CP-violating phases control the generation of the $3$ $\nu$
in the leptogenesis scenario, in $|\langle m \rangle|$ and in the leptonic CP-violating rephasing
invariant $J_{cp}$. 

2012-06-09
Theoretical Challenges

**Bi-Large mixing angles**

- **Some new flavor symmetries**
  Discrete symmetries are very useful to understand fermion mixing, especially if non-trivially broken in different sectors of the theory.

  relate Yukawa couplings of different families further reduce the number of parameters

- **A clue to the nature among quark–lepton physics beyond SM**
  Those results should be compared with the quark mixing angles in the $\nu_{\text{CKM}}$.

\[
\theta_{12}^q = (13.03 \pm 0.05)^\circ, \quad \theta_{23}^q = (2.37^{+0.05}_{-0.09})^\circ, \quad \theta_{13}^q = (0.20^{+0.02}_{-0.02})^\circ, \quad \delta_{\text{CP}}^q = (67.17^{+2.78}_{-2.44})^\circ
\]
Theoretical Challenges

What does it mean by?

- **Bi-Large mixing angles**
- **Some new flavor symmetries**
  Discrete symmetries are very useful to understand fermion mixing, especially if non-trivially broken in different sectors of the theory.
  
  relate Yukawa couplings of different families
  
  further reduce the number of parameters

- **A clue to the nature among quark–lepton physics beyond**
  Those results should be compared with the quark mixing angles.

\[
\theta_{12}^q = (13.03 \pm 0.05)^\circ, \quad \theta_{23}^q = (2.37^{+0.05}_{-0.09})^\circ, \quad \theta_{13}^q = (0.20^{+0.02}_{-0.02})^\circ
\]

\[
\theta_{12} + \theta_{12}^q = 45.0^\circ, \quad \theta_{23} + \theta_{23}^q = 45^\circ
\]

Accidental or not?
Nothing is known about all three CP–violating phases $\delta_{CP}$, $\varphi_1$, $\varphi_2$.

The relatively large $\theta_{13} \approx 9^\circ$ opens up:

- CP–violation in neutrino oscillation Exps. (T2K, NO $\nu_A$⋯)
- Matter effects can experimentally determine the type of $\nu$ mass spectrum: normal or inverted mass ordering (goal of future LBL $\nu$ oscillation Exps.)
Impact of Daya Bay and RENO

Nothing is known about all three CP-violating phases $\delta_{CP}, \varphi_1, \varphi_2$.

- The relatively large $\theta_{13} \approx 9^\circ$ opens up:
  - CP-violation in neutrino oscillation Exps. (T2K, NO $\nu A\cdots$)
  - Matter effects can experimentally determine the type of $\nu$ mass spectrum:
    - normal or inverted mass ordering (goal of future LBL $\nu$ oscillation Exps.)

- **CP violations in the lepton sector are imperative**, if the baryon asymmetry of the Universe (BAU) originated from leptogenesis scenario in the seesaw models. So any observation of the leptonic CP violation, or demonstrating that CP is not a good symmetry of the leptons, can strengthen our belief in leptogenesis.

2012-06-09
Nothing is known about all three CP-violating phases $\delta_{\text{CP}}, \phi_1, \phi_2$.

If $\delta_{\text{CP}} \neq 0$, CP is violated in $\nu$ oscillations.

$\delta_{\text{CP}}$: Not directly related to leptogenesis, but would be likely in most leptogenesis models.

Majorana phases: Neutrinoless Double beta decay $\leftrightarrow$ Leptogenesis

The relatively large $\theta_{13} \approx 9^\circ$ opens up:

- CP-violation in neutrino oscillation Exps. (T2K, NO $\nu_A$-...)
- Matter effects can experimentally determine the type of $\nu$ mass spectrum: normal or inverted mass ordering (goal of future LBL $\nu$ oscillation Exps.)

$CP$ violations in the lepton sector are imperative, if the baryon asymmetry of the Universe (BAU) originated from leptogenesis scenario in the seesaw models. So any observation of the leptonic $CP$ violation, or demonstrating that $CP$ is not a good symmetry of the leptons, can strengthen our belief in leptogenesis.
In approaches to reconstruct the high-energy physics from low-energy data, one can assume a flavor symmetry, which may reduce the unknown parameters.

Unless flavor symmetries are assumed, particle masses and mixings are generally undetermined in gauge theory:

A4, T', S4, S3 ......

global: in order to avoid the presence of a new force, the corresponding gauge bosons and their flavor violating effects

spontaneously broken at the high-energy:
  in order to prevent strong flavor violating effects

broken by a set of scalar fields that transform only under the flavor sym., for which a well-defined vacuum alignment mechanism can be constructed.
Example A: Theoretical challenges

If there exists such a flavor sym. in Nature, which could depict and explain the present data, one can imagine two natural scenario:

(i) The relative large \( \theta_{13} \) could be mainly governed by the corrections of charged lepton sector:

\[
\theta_{13} = \lambda_c \sqrt{2} \approx 9^\circ
\]

BM: P. Ramond (hep-ph/0405176)
TBM: Ahn, Cheng, Oh (arXiv: 1102.0879)
Example A: Theoretical challenges

If there exists such a flavor sym. in Nature, which could depict and explain the present data, one can imagine two natural scenario:

(i) The relative large $\theta_{13}$ could be mainly governed by the corrections of charged lepton sector:

$$\theta_{13} = \lambda c/\sqrt{2} \approx 9^\circ$$

BM: P. Ramond (hep-ph/0405176)
TBM: Ahn, Cheng, Oh (arXiv: 1102.0879)

Example: PRD83, 076012 (Feb. 2011)

In a seesaw framework with discrete A4 flavor symmetry

\[
-L^f_{\text{Yuk}} = y_\nu (\bar{L}_L N_R)_1 \bar{\eta} + \frac{1}{2} M(N_R^c N_R)_1 + \frac{1}{2} \lambda_\chi (N_R^c N_R)_3 \chi
\]

\[
+ y_e (\bar{L}_L \Phi)_1 l_R + y_\mu (\bar{L}_L \Phi)_1 l_R'' + y_\tau (\bar{L}_L \Phi)_1 l_R'''
\]

\[
+ y_u (\bar{Q}_L \tilde{\Phi})_1 u_R + y_e (\bar{Q}_L \tilde{\Phi})_1 u_R'' + y_t (\bar{Q}_L \tilde{\Phi})_1 u_R'''
\]

\[
+ y_d (\bar{Q}_L \tilde{\Phi})_1 d_R + y_s (\bar{Q}_L \tilde{\Phi})_1 d_R'' + y_b (\bar{Q}_L \tilde{\Phi})_1 d_R'''
\]

Same structure

**QLC ?**
**Example A : Theoretical challenges**

- At leading order

- **U_{CKM} = 1**
  
  **TBM(\theta_{12}=35.3^o, \theta_{23}=45, \theta_{13}=0)**

**NO LEPTOGENESIS**

- conventional works
  (E.Ma and G.Rajarasekaran; G.Altarelly and F.Feruglio; X.G.He, Y.Y.Keum and R.Volkas)

- In the light of Daya Bay and RENO results, models that lead to TBM appear to be disfavored.
  But, in all models corrections exist.

- Higher dimensional operators driven by \( \chi \) field, as equal footings

\[
\frac{y^s}{\Lambda}[(\bar{L}_L N_R)_{3s}\chi]\tilde{n}, \quad \frac{y_e}{\Lambda}[(\bar{L}_L \Phi)_{3s}\chi]\bar{l}, \quad \frac{y_d}{\Lambda}[(\bar{Q}_L \Phi)_{3s}\chi]u, \quad \frac{y_u}{\Lambda}[(\bar{Q}_L \Phi)_{3s}\chi]d, \quad \ldots
\]

increased the \# of parameters: even explain both quark and lepton part, complicated again!!
Example B : Theoretical challenges (Today Talk)

(ii) Still neutrino guys are searching for new symmetries giving easily rise to $\theta_{13} \approx 9^\circ$, while keeping $\theta_{23} \approx 45^\circ$ and $\theta_{12} \approx 35^\circ$, at leading order, not giving up the non-trivial breaking pattern of the flavor group.

Our model based on renormalizable $SU(2) \times U(1) \times A_4$ Lagrangian with minimal Yukawa coupling parameters leads to non-degenerate Dirac neutrino Yukawa coupling matrix and an unique CP source, which opens the possibility of explaining the non-zero $\theta_{13} \approx 9^\circ$ with TBM ($\theta_{23} \approx 45^\circ, \theta_{12} \approx 35^\circ$) and allowing low energy CP violation responsible for the neutrino oscillation as well as high energy CP violation responsible for leptogenesis economically. In addition, there are residual symmetries after breaking of $A_4$ flavor symmetry, which makes the lightest particle charged under this symmetry stable, providing a dark matter candidate.
A4 Symmetry (Smallest group for lepton)

A4 is the symmetry group of the tetrahedron and the finite groups of the even permutation of four objects: \(4!/2=12\) elements

**Generator** : \(S\) and \(T\) with the relations \(S^2=T^3=(ST)^3=I\)

- \(S^2=I\) \(\iff\) Z2 symmetry
- \(T^3=I\) \(\iff\) Z3 symmetry

**Elements** : I, S, T, ST, TS, \(T^2\), STS, TST, \(T^2S\), TST\(^2\), \(T^2ST\)

\[
\begin{align*}
I & \rightarrow (1,2,3,4) \\
S & \rightarrow (2,1,4,3) \\
T & \rightarrow (1,3,4,2) \\
\cdots & \cdots \\
T^2ST & \rightarrow (2,3,1,4)
\end{align*}
\]
A4 Symmetry

A4 is the symmetry group of the tetrahedron and the finite groups of the even permutation of four objects: its irreducible representations contain one triplet $3$ and three singlets $1, 1', 1''$ with the multiplication rules $3 \times 3 = 3s + 3a + 1 + 1' + 1''$ and $1' \times 1' = 1''$

The 12 representation matrices for $3$

- **Identity matrix**: $I$

- **Reflection matrices**:
  
  $r_1 = \text{diag.}(1, -1, -1), \quad r_2 = \text{diag.}(-1, 1, -1), \quad r_3 = \text{diag.}(-1, -1, 1)$

- **Cyclic, Anticyclic matrices**:

  $c = a^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad a = c^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad r_i c r_i, \quad r_i a r_i$
A4 Symmetry

A4 is the symmetry group of the tetrahedron and the finite groups of the even permutation of four objects: its irreducible representations contain one triplet $3$ and three singlets $1, 1', 1''$ with the multiplication rules $3 \times 3 = 3s + 3a + 1 + 1' + 1''$ and $1' \times 1' = 1''$

The 12 representation matrices for $3$

- **Identity matrix**: $I$

- **Reflection matrices**:
  
  $r_1 = \text{diag.}(1, -1, -1)$, $r_2 = \text{diag.}(-1, 1, -1)$, $r_3 = \text{diag.}(-1, -1, 1)$

- **Cyclic, Anticyclic matrices**:

  \[
  c = a^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}
  \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}
  \]

Subgroup

BAU\[\begin{array}{c}
\text{Z2} \\
\text{Z3}
\end{array}\]

$U_{\text{CKM}} \neq 1$

$|U_{e3}| \approx 9.2^\circ$

PRD 83, 076012 (Feb. 2011)
A4 Symmetry

Multiplication rules: \(\begin{align*}
3 \times 3 & = 3s + 3a + 1 + 1' + 1'' \quad \text{and} \quad 1' \times 1' = 1''
\end{align*}\)

In the basis where \(S\) is real diagonal

\[
S = \begin{pmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1
\end{pmatrix}, \quad T = \begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{pmatrix}
\]

Let’s denote two A4 triplets \((a_1,a_2,a_3)\) and \((b_1,b_2,b_3)\)

\[
3 \times 3 \quad 1 : (ab)_1 = a_1b_1 + a_2b_2 + a_3b_3
\]

\[
3 \times 3 \quad 1' : (ab)_1' = a_1b_1 + \omega^2a_2b_2 + \omega a_3b_3 \quad \text{where} \quad \omega = \exp(2\pi i/3)
\]

\[
\Rightarrow S: \quad a_1b_1 + \omega^2(-a_2)(-b_2) + \omega(-a_3)(-b_3)
\]

\[
\Rightarrow T: \quad a_2b_2 + \omega^2a_3b_3 + \omega a_1b_1 = \omega(a_1b_1 + \omega^2a_2b_2 + \omega a_3b_3)
\]

\[
3 \times 3 \quad 1'' : (ab)_1'' = a_1b_1 + \omega a_2b_2 + \omega^2a_3b_3
\]

\[
3 \times 3 \quad 3s : (ab)_{3s} = (a_2b_3 + a_3b_2, a_3b_1 + a_1b_3, a_1b_2 + a_2b_1) \quad (ab)_{3s} = (ba)_{3s}
\]

\[
3 \times 3 \quad 3a : (ab)_{3a} = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1) \quad (ab)_{3a} = -(ba)_{3a}
\]
We work in the framework of the SM, extended to include the right-handed $SU(2)$ singlet Majorana neutrinos $N_R$.

The scalar sector, apart from the usual SM Higgs doublet $\Phi$, is extended through the introduction of two types of scalar fields, $\chi$ and $\eta$, that are gauge singlet and doublet, respectively.

$$\Phi = (\varphi^+, \varphi^0)^T, \quad \chi, \quad \eta = (\eta^+, \eta^0)^T$$

- To understand the present non-zero $\theta_{13} \approx 9^\circ$ and TBM angles ($\theta_{12} \approx 34^\circ$, $\theta_{23} \approx 45^\circ$) in neutrino oscillation, baryogenesis and dark matter, THEORETICALLY (cf. in a radiative seesaw)

- In type-I seesaw framework, we propose a new simplest discrete symmetry based on $A_4$ flavor symmetry for leptons in renormalizable Lagrangian.

- For quarks could be promoted by extension of models based on the binary tetrahedral group $T'$ (Frampton, Kephart 1995)
Construction of Lagrangian

Under $SU(2) \times U(1) \times A_4$

<table>
<thead>
<tr>
<th>Field</th>
<th>$L_e, L_\mu, L_\tau$</th>
<th>$l_R, l'_R, l''_R$</th>
<th>$N_R$</th>
<th>$\chi$</th>
<th>$\Phi$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_4$</td>
<td>1, 1', 1''</td>
<td>1, 1', 1''</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$SU(2)_L \times U(1)_Y$</td>
<td>(2, $-\frac{1}{2}$)</td>
<td>(1, −1)</td>
<td>(1, 0)</td>
<td>(1, 0)</td>
<td>(2, $\frac{1}{2}$)</td>
<td>(2, $\frac{1}{2}$)</td>
</tr>
</tbody>
</table>

The renormalizable Yukawa interactions in the neutrino and charged lepton sectors

\[-\mathcal{L}_{\text{Yuk}} = y'_1 \bar{L}_e (\tilde{\eta} N_R)_1 + y'_2 \bar{L}_\mu (\tilde{\eta} N_R)_1' + y'_3 \bar{L}_\tau (\tilde{\eta} N_R)_1''
+ \frac{1}{2} M (\bar{N}_R^c N_R)_1 + \frac{1}{2} \lambda_\chi (\bar{N}_R^c N_R)_3 \chi
+ y_e \bar{L}_e \Phi l_R + y_\mu \bar{L}_\mu \Phi l'_R + y_\tau \bar{L}_\tau \Phi l''_R + h.c.\]

- Each Dirac-$\nu$ and charged-lepton sector has three independent Yukawa terms
- Heavy $\nu$s acquire a bare mass term $M$ and a mass induced by $\chi$
- The three leptons $e, \mu, \tau$ are eigenstates of $T$ with eigenvalues 1, $\omega$, $\omega^2$, respectively

\[L_e \sim 1 \quad L_\mu \sim \omega \quad L_\tau \sim \omega^2\]
\[e_R \sim 1 \quad \mu_R \sim \omega \quad \tau_R \sim \omega^2\]

As a consequence, the charged lepton mass matrix automatically diagonal.

2012-06-09
If the flavor symmetry associated to A4 is broken by the VEV of a triplet \( \eta \) and \( \chi \) of scalar fields, there are non-trivial breaking pattern:

\[
<\eta>=(1,1,1) : A4 \rightarrow \mathbb{Z}_3 , \quad <\chi>=(1,0,0) : A4 \rightarrow \mathbb{Z}_2
\]

A4 completely broken in the whole theory

Under \( SU(2) \times U(1) \times A4 \), the most general renormalizable scalar potential of \( \eta \) and \( \chi \) :

\[
V(\eta) = \mu_\eta^2 (\eta^\dagger \eta)_1 + \lambda^\eta_1 (\eta^\dagger \eta)_1 (\eta^\dagger \eta)_1 + \lambda^\eta_2 (\eta^\dagger \eta)(\eta^\dagger \eta)_1 + \lambda^\eta_3 (\eta^\dagger \eta)_3 (\eta^\dagger \eta)_3
\]
\[
+ \lambda^\eta_4 (\eta^\dagger \eta)_3 (\eta^\dagger \eta)_3 + \{\lambda^\eta_5 (\eta^\dagger \eta)_3 (\eta^\dagger \eta)_3 + h.c\} ,
\]

\[
V(\Phi) = \mu_\Phi^2 (\Phi^\dagger \Phi) + \lambda^\Phi (\Phi^\dagger \Phi)^2 ,
\]

\[
V(\chi) = \mu_\chi^2 (\chi \chi)_1 + \lambda^\chi_1 (\chi \chi)_1 (\chi \chi)_1 + \lambda^\chi_2 (\chi \chi)(\chi \chi)_1 + \lambda^\chi_3 (\chi \chi)_3 (\chi \chi)_3
\]
\[
+ \lambda^\chi_4 (\chi \chi)_3 (\chi \chi)_3 + \lambda^\chi_5 (\chi \chi)_3 (\chi \chi)_3 + \xi_1 \chi (\chi \chi)_3 + \xi_2 \chi (\chi \chi)_3 ,
\]

\[
V(\eta \Phi) = \lambda^{n\Phi}_1 (\eta^\dagger \eta)_1 (\Phi^\dagger \Phi) + \lambda^{n\Phi}_2 (\eta^\dagger \eta)(\Phi^\dagger \Phi) + \{\lambda^{n\Phi}_3 (\eta^\dagger \eta)(\eta^\dagger \Phi) + h.c\}
\]
\[
+ \{\lambda^{n\Phi}_4 (\eta^\dagger \eta)_3 (\eta^\dagger \Phi) + h.c\} + \{\lambda^{n\Phi}_5 (\eta^\dagger \eta)_3 (\eta^\dagger \Phi) + h.c\} ,
\]

\[+ V(\eta \chi)\]

In the presence of two A4–triplet Higgs scalars, in vacuum stability, Problematic interaction term \( V(\eta \chi) \)
If the flavor symmetry associated to A4 is broken by the VEV of a triplet $\eta$ and $\chi$ of scalar fields, there are non-trivial breaking pattern:

\[ \langle \eta \rangle = (1,1,1) : A4 \rightarrow Z_3 \, , \quad \langle \chi \rangle = (1,0,0) : A4 \rightarrow Z_2 \]

A4 completely broken in the whole theory.

Under $SU(2) \times U(1) \times A4$, the most general renormalizable scalar potential of $\eta$ and $\chi$:

\[
V_{y=L} = V(\Phi) + V(\eta) + V(\eta \Phi)
\]

\[
V_{y=0} = V(\chi)
\]

can be naturally solved by

In the presence of two A4–triplet Higgs scalars, in vacuum stability, Problematic interaction term $V(\eta \chi)$
After EW symmetry breaking and the minimization of the potential

\[ \Phi = \begin{pmatrix} \varphi^+ \\ \frac{1}{\sqrt{2}} (v\phi + h + iA_0) \end{pmatrix}, \quad \chi_1 = v\chi + \chi_1^0, \quad \chi_2 = \chi_2^0, \quad \chi_3 = \chi_3^0 \]

\[ \eta_j = \begin{pmatrix} \eta_j^+ \\ \frac{1}{\sqrt{2}} (v\eta + h_j + iA_j) \end{pmatrix}, \quad (j = 1, 2, 3) \]

Doublet VEV is symmetric under T:

\[ \begin{pmatrix} v_n/\sqrt{3} \\ v_n/\sqrt{3} \\ v_n/\sqrt{3} \end{pmatrix} \xrightarrow{T} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_n/\sqrt{3} \\ v_n/\sqrt{3} \\ v_n/\sqrt{3} \end{pmatrix} = \begin{pmatrix} v_n/\sqrt{3} \\ v_n/\sqrt{3} \\ v_n/\sqrt{3} \end{pmatrix} \]

A4 $\rightarrow$ Z3 sym.

Singlet VEV is symmetric under S:

\[ \begin{pmatrix} v_\chi \\ 0 \\ 0 \end{pmatrix} \xrightarrow{S} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} v_\chi \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} v_\chi \\ 0 \\ 0 \end{pmatrix} \]

A4 $\rightarrow$ Z2 sym.
Residual symmetry after A4 breaking

Once the VEV alignments is taken, the A4 sym. is spontaneously broken to its residual sym. Z3 in the Dirac neutrino sector, while A4 broken to its residual sym. Z2 in the heavy neutrino sector.

\[-\mathcal{L}_m = \frac{v_{\Phi}}{\sqrt{2}} (y_{e} e_L e_R + y_{\mu} \bar{\mu}_L \mu_R + y_{\tau} \bar{\tau}_L \tau_R) + \frac{v_{\eta}}{\sqrt{2}} \left\{ (y_{1}^\nu \bar{\nu}_e + y_{2}^\nu \bar{\nu}_\mu + y_{3}^\nu \bar{\nu}_\tau) N_{R1} \right. \\
+ \left. (y_{1}^{\nu} \bar{\nu}_e + y_{2}^{\nu} \omega \bar{\nu}_\mu + y_{3}^{\nu} \omega^2 \bar{\nu}_\tau) N_{R2} + (y_{1}^{\nu} \bar{\nu}_e + y_{2}^{\nu} \omega^2 \bar{\nu}_\mu + y_{3}^{\nu} \omega \bar{\nu}_\tau) N_{R3} \right\} \\
+ \frac{M}{2} (\bar{N}_{c}^c N_{R1} + \bar{N}_{c}^c N_{R2} + \bar{N}_{c}^c N_{R3}) + \frac{\lambda_{\chi} v_{\chi}}{2} (\bar{N}_{c}^c N_{R3} + \bar{N}_{c}^c N_{R2}) + h.c.
\]

All the particles can be classified by Z3 charge !

Singlets 1 : $\nu_e(e) \rightarrow \nu_e(e)$

1' : $\nu_\mu(\mu) \rightarrow \omega \nu_\mu(\omega \mu)$

1'' : $\nu_\tau(\tau) \rightarrow \omega^2 \nu_\tau(\omega^2 \tau)$

Triplets 3 : $\Psi = N_R, \chi, A_i, h_i$

\[
\begin{pmatrix}
\Psi'_1 \\
\Psi'_2 \\
\Psi'_3
\end{pmatrix} = \frac{1}{\sqrt{3}}
\begin{pmatrix}
1 & 1 & 1 \\
1 & \omega & \omega \\
1 & \omega & \omega
\end{pmatrix}
\begin{pmatrix}
\Psi_1 \\
\Psi_2 \\
\Psi_3
\end{pmatrix}
\]

$\sqrt{3} \Psi'_1 = \Psi_1 + \Psi_2 + \Psi_3 \rightarrow T \Psi_2 + \Psi_3 + \Psi_1 = \sqrt{3} \Psi'_1$

$\sqrt{3} \Psi'_2 = \Psi_1 + \omega^2 \Psi_2 + \omega \Psi_3 \rightarrow T \Psi_2 + \omega^2 \Psi_3 + \omega \Psi_1 = \omega \sqrt{3} \Psi'_2$

$\sqrt{3} \Psi'_3 = \Psi_1 + \omega \Psi_2 + \omega^2 \Psi_3 \rightarrow T \Psi_2 + \omega \Psi_3 + \omega^2 \Psi_1 = \omega^2 \sqrt{3} \Psi'_3$
Residual symmetry after A4 breaking

- Until now we have not seen the Z3 sym. in Nature.
- The stability of the DM can be guaranteed by the Z3 parity in Yukawa Dirac neutrino and Higgs sectors.

All the particles can be classified by Z3 charge!!

\[
egin{align*}
N'_{R1} & \rightarrow N'_{R1} & h'_1(A'_1) & \rightarrow h'_1(A'_1) & \nu_e(e) & \rightarrow \nu_e(e) \\
N'_{R2} & \rightarrow \omega N'_{R2} & h'_2(A'_2) & \rightarrow \omega h'_2(\omega A'_2) & \nu_\mu(\mu) & \rightarrow \omega \nu_\mu(\omega \mu) \\
N'_{R3} & \rightarrow \omega^2 N'_{R3} & h'_3(A'_3) & \rightarrow \omega^2 h'_3(\omega^2 A'_3) & \nu_\tau(\tau) & \rightarrow \omega^2 \nu_\tau(\omega^2 \tau)
\end{align*}
\]

DM candidate: the lightest combination of prime\{h_2, h_3, A_2 and A_3 \}

The DM candidate (i) only couples to the heavy neutrinos and not to the SM charged fermions.

(ii) can interact with the standard Higgs boson, h

(iii) can interact with the gauge bosons W,Z

Working in progress
In a weak eigenstate basis

\[ -\mathcal{L} = \frac{1}{2} N_R^c M_R N_R + \bar{\ell}_L m_\ell \ell_R + \bar{\nu}_L m_D N_R + \frac{g}{\sqrt{2}} W_\mu \bar{\ell}_L \gamma^\mu \nu_L + h.c., \]

where \( m_\ell = \frac{v_e}{\sqrt{2}} \text{Diag.}(y_e, y_\mu, y_\tau) \) and \( m_D = v_\eta Y_\nu / \sqrt{2} \).

Performing basis rotations from weak eigenstate to mass eigenstate

\[ N_R \rightarrow U_R^\dagger N_R, \quad \ell_L \rightarrow P_\ell^* \ell_L, \quad \ell_R \rightarrow P_\ell^* \ell_R, \quad \nu_L \rightarrow U_\nu^\dagger P_\nu^* \nu_L \]

\[ \hat{M}_R = U_R^T M_R U_R = M U_R^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \kappa e^{i\xi} \\ 0 & \kappa e^{i\xi} & 1 \end{pmatrix} U_R = M \text{Diag.}(a, 1, b) \]

\[ Y_\nu \rightarrow \hat{Y}_\nu = Y_\nu U_\omega U_R = y_3 \sqrt{3} \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} U_\omega U_R \]
Residual symmetry after A4 breaking

In a weak eigenstate basis

\[
-\mathcal{L} = \frac{1}{2} N_R C M_R N_R + \overline{\ell}_L m_\ell \ell_R + \overline{\nu}_L m_D N_R + \frac{g}{\sqrt{2}} \overline{W}_\mu \ell_L \gamma^\mu \nu_L + h.c.
\]

where \( m_\ell = \frac{v_f}{\sqrt{2}} \text{Diag.}(y_e, y_\mu, y_\tau) \) and \( m_D = v_\eta Y_\nu / \sqrt{2} \).

Performing basis rotations from weak eigenstate to mass eigenstate

\[
\begin{align*}
N_R &\rightarrow U_R^\dagger N_R \\
\ell_L &\rightarrow P_\ell^* \ell_L,
\ell_R &\rightarrow P_\ell^* \ell_R,
\nu_L &\rightarrow U_\nu^* P_\nu^* \nu_L
\end{align*}
\]

\( M_R = U_R^T M_R U_R = M U_R^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \kappa e^{i\xi} \\ 0 & \kappa e^{i\xi} & 1 \end{pmatrix} \)

\( U_R = M \text{Diag.}(a, 1, b) \)

\( Y_\nu \rightarrow \tilde{Y}_\nu = Y_\nu U_\omega U_R = y_3 \sqrt{3} \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \)

Bi-Maximal

Tri-Maximal

Non-zero \( \theta_{13} \) & high-energy CP violation
Residual symmetry after A4 breaking

From the charged current term: Three mixing angles and Three CP-odd phases

\[ U_{\text{PMNS}} = P^* \rho U_\nu = U_\nu \]

\[
\begin{pmatrix}
  c_{13} c_{12} & c_{13} s_{12} & s_{13} e^{-i\delta_{CP}} \\
  -c_{23} s_{12} - s_{23} c_{12} s_{13} e^{i\delta_{CP}} & c_{23} c_{12} - s_{23} s_{12} s_{13} e^{i\delta_{CP}} & s_{23} c_{13} \\
  s_{23} s_{12} - c_{23} c_{12} s_{13} e^{i\delta_{CP}} & -s_{23} c_{12} - c_{23} s_{12} s_{13} e^{i\delta_{CP}} & c_{23} c_{13}
\end{pmatrix} Q_\nu
\]

Phase matrix \( P_\nu \) comes from Dirac neutrino coupling matrix, which can be rotated away by the redefinition of the left-charged lepton fields

\( Q_\nu \) is Maj. phase matrix

CP-Asymmetry relevant for leptogenesis is associated with the neutrino Yukawa coupling matrix itself and its combination

\[ H = |y_3'|^2 \begin{pmatrix}
\frac{1+4y_1^2+y_2^2}{2} & \frac{e^{-i\frac{\psi_1}{2}}}{\sqrt{2}} (2y_1^2 - y_2^2 - 1) & -i\sqrt{3}e^{i\frac{\psi_2}{2}} \frac{1}{2} (y_2^2 - 1) \\
\frac{e^{i\frac{\psi_2}{2}}}{\sqrt{2}} (2y_1^2 - y_2^2 - 1) & 1 + y_1^2 + y_2^2 & \frac{1}{2} \sqrt{3} e^{-i\frac{\psi_2}{2}} (y_2^2 - 1) \\
i\sqrt{3}e^{-i\frac{\psi_1}{2}} \frac{1}{2} (y_2^2 - 1) & -i\sqrt{3}e^{i\frac{\psi_2}{2}} \frac{1}{2} (y_2^2 - 1) & \frac{3}{2} (1 + y_2^2)
\end{pmatrix}
\]

Non-zero entries and imaginaries: deviation \( y_1, y_2 \) from unit, \( \psi_1, \psi_2 (\kappa, \xi) \)

2012-06-09
After seesawing, in a basis where charged lepton and heavy neutrino masses are real and diagonal

\[
m_{\nu} = -\tilde{m}_D \tilde{M}_R^{-1} \tilde{m}_D^T = -\frac{\nu^2}{2} Y_\nu U_\omega U_R \tilde{M}_R^{-1} U_R^T Y_\nu^T
\]

\[
= \begin{pmatrix}
(1 + \frac{2e^{i\psi_1}}{a}) y_1^2 & (1 - \frac{e^{i\psi_1}}{a}) y_1 y_2 & (1 - \frac{e^{i\psi_1}}{a}) y_1 \\
(1 - \frac{e^{i\psi_1}}{a}) y_1 y_2 & (1 + \frac{e^{i\psi_1}}{2a} + \frac{3e^{i\psi_2}}{2b}) y_2^2 & (1 + \frac{e^{i\psi_1}}{2a} - \frac{3e^{i\psi_2}}{2b}) y_2 \\
(1 - \frac{e^{i\psi_1}}{a}) y_1 & (1 + \frac{e^{i\psi_1}}{2a} - \frac{3e^{i\psi_2}}{2b}) y_2 & (1 + \frac{e^{i\psi_1}}{2a} + \frac{3e^{i\psi_2}}{2b})
\end{pmatrix}
\]

\[
m_{\nu} = U_{PMNS} \text{ Diag}(m_1, m_2, m_3) U_{PMNS}^T
\]

\[
U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix}
1 & 1 & 1 \\
1 & \omega^2 & \omega \\
1 & \omega & \omega^2
\end{pmatrix}
\]

\[
U_R = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & \sqrt{2} & \frac{e^{i\psi_1}}{2} & 0 & 0 \\
1 & 0 & -1 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 \\
0 & 0 & e^{i\psi_2} & 0 & 1
\end{pmatrix}
\]
After seesawing, in a basis where charged lepton and heavy neutrino masses are real and diagonal

\[
m_{\nu} = -\tilde{m}_D \hat{M}_R^{-1} m_D = -\frac{\nu^2}{2} Y_{\nu} U_\omega U_R \hat{M}_R^{-1} U_R^T U_\omega^T Y_{\nu}^T
\]

\[
= m_0 \begin{pmatrix}
(1 + \frac{2e^{i\psi_1}}{a})y_1^2 & (1 - \frac{e^{i\psi_1}}{a})y_1y_2 & (1 - \frac{e^{i\psi_1}}{a})y_1 \\
(1 - \frac{e^{i\psi_1}}{a})y_1y_2 & (1 + \frac{e^{i\psi_1}}{2a} + \frac{3e^{i\psi_2}}{2b})y_2^2 & (1 + \frac{e^{i\psi_1}}{2a} - \frac{3e^{i\psi_2}}{2b})y_2 \\
(1 - \frac{e^{i\psi_1}}{a})y_1 & (1 + \frac{e^{i\psi_1}}{2a} - \frac{3e^{i\psi_2}}{2b})y_2 & (1 + \frac{e^{i\psi_1}}{2a} + \frac{3e^{i\psi_2}}{2b})
\end{pmatrix}
\]

\[
m_{\nu} = U_{PMNS} \text{ Diag}(m_1, m_2, m_3) U_{PMNS}^T
\]

\[
U_\omega U_R = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\end{pmatrix} \begin{pmatrix}
e^{\frac{i\psi_1}{2}} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & e^{\frac{i\psi_2 + \pi}{2}}
\end{pmatrix}
\]

\[
Y_{\nu} \rightarrow I \text{ (i.e. } y_1, y_2 \rightarrow 1) : \text{ it is clear that the exact TBM is recovered !!}
\]

\[
\text{Recent neutrino data, } \theta_{13} \neq 0, \text{ require the deviations } y_1 \text{ and } y_2 \text{ from unit, leading to a possibility for searching for CP–violation in neutrino oscillation experiments}
\]

2012-06-09
Phenomenology in low energy Neutrino

Deviations from the TBM

\[ \theta_{23} = -\frac{\pi}{4} + \epsilon_1, \quad \theta_{13} = \epsilon_2, \quad \theta_{12} = \sin^{-1} \left( \frac{1}{\sqrt{3}} \right) + \epsilon_3 \]

or equivalently

\[
U_{\text{PMNS}} = \begin{pmatrix}
\frac{\sqrt{2}-\epsilon_3}{\sqrt{3}} & \frac{1+\epsilon_3\sqrt{2}}{\sqrt{3}} & \epsilon_2 e^{-i\delta_{CP}} \\
-\frac{1+\epsilon_1+\epsilon_3\sqrt{2}}{\sqrt{6}} + \frac{\epsilon_2 e^{i\delta_{CP}}}{\sqrt{3}} & \frac{\sqrt{2}+\epsilon_1\sqrt{2}-\epsilon_3}{\sqrt{3}} + \frac{\epsilon_2 e^{i\delta_{CP}}}{\sqrt{6}} & \frac{-1+\epsilon_1}{\sqrt{2}} \\
-\frac{1+\epsilon_1+\epsilon_3\sqrt{2}}{\sqrt{6}} - \frac{\epsilon_2 e^{i\delta_{CP}}}{\sqrt{3}} & \frac{\sqrt{2}+\epsilon_1\sqrt{2}-\epsilon_3}{\sqrt{6}} + \frac{\epsilon_2 e^{i\delta_{CP}}}{\sqrt{6}} & \frac{1+\epsilon_1}{\sqrt{2}}
\end{pmatrix} Q_\nu + \mathcal{O}(\epsilon_1^2)
\]

\[
\tan \epsilon_1 = \frac{R(y_2 - 1) - S(y_2 - 1)}{R(y_2 - 1) - S(1 + y_2)}
\]

\[
\tan 2\theta_{13} = \frac{y_1|s_{23}(3Q - P)y_2 - c_{23}(3Q + P) - 3is_{23}(R - S)y_2 + c_{23}(R + S)|}{(F + G + \frac{9K}{4} + \frac{3D}{2})(c_{23}^2 + \frac{y_2^2 s_{23}^2}{y_1})(\sin 2\theta_{23} - y_1^2 \tilde{A})}
\]

\[
\tan \delta_{CP} = 3\frac{(R - S)^2 + y_2^2(R + S)^2}{(P + Q)(R - S) - y_2^2(P - Q)(R + S)}
\]

- In the limit of \(y_1, y_2, \epsilon_1 & \epsilon_2\) go to zero due to R,S, Q→ 0

- Here P,Q,R⋯ are the compts. of the hermitian matrix

\[
m_\nu m_\nu^\dagger = U_{\text{PMNS}} \text{Diag.}(m_1^2, m_2^2, m_3^2) U_{\text{PMNS}}^\dagger
\]
Phenomenology in low energy Neutrino Solar mixing angle

\[ \tan 2\theta_{12} = 2y_1 \frac{y_2 s_{23} (3Q - P) + s_{23} (3Q + P)}{c_{13} (\Psi_2 - \Psi_1)} \]

- In the limit of \( y_1, y_2 \rightarrow 0 \), it is clear that \( \tan 2\theta_{12} \rightarrow 2\sqrt{2} \)
due to \( Q \rightarrow 0, P \rightarrow 6(1/a^2 - 1), \Psi_1 \rightarrow 3(1+2/a^2) \) and \( \Psi_2 \rightarrow 6(1+1/2a^2) \)

Leptonic CP–violation at low–energies can be detected through neutrino oscillations which are sensitive to the Dirac–phase, but insensitive to the Majorana phases.

\[ J_{CP} \equiv \text{Im}[U_{e1} U_{\mu2} U_{e2}^* U_{\mu1}^*] = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta_{CP} \]

\[ = - \frac{\text{Im}\{h_{12} h_{23} h_{31}\}}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2} \]

where \( h = m_\nu m_\nu^\dagger \)

- In the limit of \( y_2 \rightarrow 0 \) or \( \sin \psi_2 \rightarrow 0 \), \( J_{CP} \) goes to zero
Mixing Angles

Leptonic CP violation Vs Reactor angle $\theta_{13}$

Dirac–phase $\delta_{CP}$ Vs Atm. angle $\theta_{23}$
Mixing Angles

Atm. angle $\theta_{23}$ Vs Reactor angle $\theta_{13}$

Solar angle $\theta_{12}$ Vs Reactor angle $\theta_{13}$

2012-06-09
Neutrinoless Double-beta Decay

$|m_{ee}|$ vs $\theta_{13}$

$|m_{ee}|$ vs $m_{\text{lightest}}$
Conclusions and discussions

- We have found a simple model based on A4 flavor symmetry in a type-I seesaw mechanism, which seems to be very attractive because of their predictions of TBM angles and non-zero $\theta_{13}$ as well as a natural source of low and high energy CP violation for neutrino oscillation and leptogenesis, respectively, and dark matter in renormalizable Lagrangian.

- All the parameters are assumed to be real $\Rightarrow$ CP-invariance

Once the single $\chi$ acquires a complex VEV, the CP symmetry is spontaneously broken at high energies by the complex VEV of the scalar singlet. Such a breaking leads to leptonic CP violation at low energies, see $\psi_2$. 

2012-06-09