Endpoint Logarithms in

\[ e^+ e^- \rightarrow J/\psi + \eta_c \]

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Outline

- $e^+e^- \rightarrow J/\psi + \eta_c$ in B factories
- Endpoint double logarithms in $e^+e^- \rightarrow J/\psi + \eta_c$
- Summary
Exclusive Double Charmonium Production

- Belle (2004): \( \sigma[e^+e^- \rightarrow J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4 \text{fb} \)

- BABAR (2005): \( \sigma[e^+e^- \rightarrow J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1} \text{fb} \)
  BABAR, PRD72, 031101(R) (2005)

![Graph showing exclusive double charmonium production spectrum.](image)
Exclusive Double Charmonium Production

• Factorized form of production amplitude
  \[ \mathcal{A} = J^- \otimes H \otimes J^+ \]

• \( J^\pm \) are non-perturbative meson distribution amplitudes. These contain all long-distance physics that corresponds to the evolution from \( Q\bar{Q} \) pairs to mesons.

• \( H \) depends only contributions from short distances, and can be calculated using perturbation theory.

• Radiative corrections may involve logarithms in \( m^2/s \), which can make a fixed-order calculation unreliable. Factorization theorem allows us to resum such logarithms.
Helicity Selection Rule

• Asymptotic behavior for \( s \gg m^2 \) is given by the helicity selection rule

\[
\sigma(e^+e^- \to A + B) \sim \frac{(m^2/s)^{2+|\lambda_A+\lambda_B|}}{s}
\]

That is, processes preserving hadron helicity have slowest asymptotic decrease.  

Brodsky and Lepage, PRD24, 2848 (1981)

• In \( e^+e^- \to J/\psi + \eta_c \), \( J/\psi \) is produced with helicity \( \pm 1 \) 
(zero helicity forbidden by parity invariance)

\[
\sigma(e^+e^- \to J/\psi + \eta_c) \sim \frac{(m^2/s)^3}{s}
\]

The suppression \( m^2/s \) comes from quark helicity flip.
Helicity Selection Rule


- NLO correction to helicity-preserving processes involve single logarithm $\alpha_s \log(m^2/s)$, which can be resummed using evolution equations. Efremov and Radyushkin, PLB94, 245 (1980) Lepage and Brodsky, PRD22, 2157 (1980)

- In contrast, factorization theorem for helicity-suppressed processes is unavailable.

- NLO corrections to helicity-suppressed processes are known to involve double logarithms $\alpha_s \log^2(m^2/s)$ (In B factories, $\alpha_s \log^2(m^2/s) \approx 3$), resummation is necessary. Jia, Wang and Yang, JHEP10, 105 (2011)
Double Logarithms in $e^+ e^- \rightarrow J/\psi + \eta_c$

- Factorization theorem allows one to resum large logarithms.
- A first step in proving factorization is the identification of the momentum regions that give rise to large logarithms.
- In this work we investigate the origin of the double logarithms at NLO in $\alpha_s$ by explicit evaluation of the NLO diagrams.
Finding Double Logarithms in
\[ e^+ e^- \rightarrow J/\psi + \eta_c \]

- We work at leading order in the relative velocity of \( Q\bar{Q} \) of the charmonia.

\[
P_{J/\psi} = 2p, \quad P_{\eta_c} = 2\bar{p}, \quad p^2 = \bar{p}^2 = m^2, \quad Q = P_{J/\psi} + P_{\eta_c}
\]

- \( p \) is collinear to plus, \( \bar{p} \) is collinear to minus.

- The logarithms in \( m \) can be identified as would-be soft and collinear singularities regulated by the quark mass \( m \).

- The process requires a quark helicity flip, so that we cannot ignore the quark masses in the numerators.
1-Loop Diagrams

- The $Q\bar{Q}$ pair going up forms the $J/\psi$, the rest forms $\eta_c$.
- There are also charge conjugation diagrams.
- Diagrams that do not allow logarithmic power counting are ignored.
Origins of Double Logarithms

• The double logarithms in each diagram come from the Sudakov and endpoint logarithms.

• Sudakov double logs: the momentum of the gluon is simultaneously soft and collinear.

• Endpoint double logs: gluons carry almost all of the collinear momentum from a spectator line to an active line. The double log occurs when the momentum of the spectator line is simultaneously soft and collinear.
Origins of Double Logarithms

- A change of variables makes the endpoint double log look like a Sudakov double log.

**Sudakov Double Logs**

\[
S = \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 + i\varepsilon)[(k - p)^2 - m^2 + i\varepsilon][(k + \bar{p})^2 - m^2 + i\varepsilon]}
\]

\[
= \frac{i}{4\pi^2 Q^2} \left[ \left( \frac{1}{\varepsilon_{\text{IR}}} - \log(m^2/\mu^2) \right) \log(m^2/Q^2) + \frac{1}{2} \log^2(m^2/Q^2) + \cdots \right]
\]

**Endpoint Double Logs**

\[
\mathcal{E} = \int \frac{d^d \ell}{(2\pi)^d} \frac{1}{(\ell^2 - m^2 + i\varepsilon)[(\ell + p)^2 + i\varepsilon][(\ell - \bar{p})^2 + i\varepsilon]}
\]

\[
= \frac{i}{8\pi^2 Q^2} [\log^2(m^2/Q^2) + \cdots].
\]
By applying the soft approximation we can prove that the Sudakov logarithms cancel between soft gluon insertions into quark and antiquark lines.

\[ S \text{ cancel in the sum} \]

\[ + \quad = \quad 0 \]

... and so on.
Sudakov Double Logarithms

- Therefore the *Sudakov double logarithms cancel in the sum of all diagrams.*

\[ e^+ e^- \rightarrow J/\psi + \eta_c \]

- The double logarithms in \( e^+ e^- \rightarrow J/\psi + \eta_c \) originate solely from endpoint double logarithms.
Endpoint Double Logarithms

- Endpoint double logs appear when $\ell$ is soft-collinear.

- Helicity flip in the spectator quark line eliminates a factor of $\ell$ in the numerator, giving logarithmic power counting for soft-collinear scaling.

\[
\mathcal{E} = \int \frac{d^d \ell}{(2\pi)^d (\ell^2 - m^2 + i\varepsilon)[(\ell + p)^2 + i\varepsilon][(\ell - \bar{p})^2 + i\varepsilon]} \frac{1}{(2\pi)^d (\ell^2 - m^2 + i\varepsilon)[(\ell + p)^2 + i\varepsilon][(\ell - \bar{p})^2 + i\varepsilon]}
\]
The following diagrams potentially contain endpoint logs.

- Soft quark line
- Collinear quark line with momentum containing $p$, but not $\bar{p}$
- Collinear quark line with momentum containing $\bar{p}$, but not $p$
Absence of Power Divergences

- The additional collinear lines may give power divergences.

- If $\ell$ is collinear to $+$, then all momenta surrounding the upper gluon is collinear to $+$. Then, the numerator vanishes because we can anticommute away any of the momenta so that it is adjacent to another one collinear to the same direction. The numerator must have a factor $\ell^-$ or $\ell_\perp$.

- Therefore the numerator must have a factor $\ell \cdot p$ or $\ell^2$. $\ell \cdot p$ produces an endpoint double log, and $\ell^2$ gives a collinear log, but not a double log.

- The same analysis can be repeated for the remaining diagrams.
Absence of Endpoint Logs without Helicity Flip

• Consider a helicity-preserving process, such as \( e^+ e^- \rightarrow h_c + \eta_c \). In the soft approximation,

\[
\int \frac{d^d \ell \ l}{(2\pi)^d \ l^2 + i\epsilon \ 2\ell \cdot p + i\epsilon - 2\ell \cdot \bar{p} + i\epsilon} \times \text{collinear fermion lines}
\]

• The collinear fermion lines give

\[
\frac{\ell + a\phi}{2a\ell \cdot p + i\epsilon}, \quad \frac{\ell + b\bar{\phi}}{2b\ell \cdot \bar{p} + i\epsilon}
\]

The \( \phi \) or \( \bar{\phi} \) is eliminated by the equations of motion; the collinear fermion lines do not change the power of the loop momentum in the integrand, giving \( \sim \int \ell 1/\ell^3 \).

Therefore **endpoint logs are absent without helicity flip**.

• This even holds when the relative velocity of \( Q\bar{Q} \) is retained.
Endpoint Logs with Helicity Flip

- In the helicity-suppressed processes, we have
  \[
  \int \frac{d^d \ell}{(2\pi)^d} \frac{\ell + m}{\ell^2 + i\varepsilon} \frac{1}{2\ell \cdot p + i\varepsilon} \frac{1}{-2\ell \cdot \bar{p} + i\varepsilon} \times \text{collinear fermion lines}
  \]

- The collinear fermion lines give
  \[
  \frac{\ell + a\bar{p} + m}{2a\ell \cdot p + i\varepsilon} \quad \frac{\ell + b\bar{p} + m}{2b\ell \cdot \bar{p} + i\varepsilon}
  \]

- Helicity flip is obtained by either picking up the quark masses or using the equations of motion for the collinear fermion lines. This eliminates a factor of \( \ell \) in the numerator.

*Helicity flip gives the correct logarithmic scaling for endpoint double logarithm.*
Summary

- We investigated the origin of the double logarithms in \( e^+ e^- \rightarrow J/\psi + \eta_c \).
- Sudakov double logarithms cancel in the sum over all diagrams, while endpoint double logarithms remain.
- The endpoint double logarithms are re-interpreted as a leading region of loop integration in which a spectator fermion line becomes soft-collinear.
- This re-interpretation may simplify the resummation of logarithms in \( m^2/s \).
- The factorization for the helicity-flip process is currently under development.
Supplementary
1-Loop Diagrams

- The following diagrams do not have double logs

+ charge-conjugation diagrams
+ diagrams with counterterms
Cancellation of Sudakov Logs

Insertion of soft gluons into collinear-to-minus lines (contribution from collinear-to-plus region).

The Sudakov double logs cancel in these combinations.

\[ + = 0 \]

\[ + = 0 \]

\[ + = 0 \]

\[ + = 0 \]
Cancellation of Sudakov Logs

Insertion of soft gluons into collinear-to-plus lines (contribution from collinear-to-minus region).
The Sudakov double logs cancel in these combinations.
Cancellation of Sudakov Logs

Because the lower meson has $C = +1$, the paired diagrams are same. The Sudakov double logarithms cancel in each diagram.