The New Minimal Supersymmetric SO(10) GUT

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Outline

Introduction: Grand Unification

Supersymmetric SO(10) GUT

MSGUT: Minimal Susy GUT

MSGUT SEESAW Failure

New Minimal Susy GUT

Fermion Fitting: NMSGUT

Summary
Standard Model (SM) is a renormalizable, spontaneously broken QFT describing the strong and electroweak interactions.

1. Gauge Group:
   \[ SU(3) \times SU(2)_L \times U(1)_Y \]

2. Higgs field (generation of Masses)

- Symmetry Breaking: \[ SU(3) \times SU(2)_L \times U(1)_Y \Rightarrow SU(3) \times U(1)_Q \]
- Explains all the low energy data successfully so far observed.
Looking Beyond SM

SM - effective field theory i.e. low energy limit of more fundamental theory. Look at various extensions (GUT, susy, extra dimensions..etc..)

- **Strong Gravity:** $M_P \sim 10^{19} \text{Gev}$
- **Gauge Coupling unification:** $M_{GUT} \sim 10^{16} \text{Gev}$
- **Neutrino Oscillations:** No $\nu_R$

Seesaw:

$$M_\nu \sim Y \frac{\nu^2}{M} \Rightarrow M \sim 10^{14\pm1} \text{GeV} \approx M_{GUT}!!$$

- **Structural instability in SM:** Corrections to Higgs Mass

Higgs Corrections: $\Delta m_H^2 \sim \frac{y_t^2 M^2}{16P_{i2}}$

If $M \sim 10^{16}\text{Gev}$ then $\Delta m_H^2 \sim O(M^2)$

- **Recent LHC Higgs Signal** $\sim 125$ GeV
  
  Excess of events in $\gamma\gamma, ZZ^*, WW^*$ channels
Grand Unification

One beautiful idea is Grand unification.

- Idea Grand Unification: J Pati and A. Salam
  \[ SU(3) \times SU(2)_L \times U(1)_Y \subset SU(4) \times SU(2)_L \times SU(2)_R \]

- SU(5) : H.Georgi and S.Glashow
  \[ SU(5) \rightarrow SM \rightarrow SU(3)_c \times U(1)_Q \]
  Strong, Weak and em Unification (PRL. 32, 438(1974))

- SO(10): H.Fritzsch and P. Minkowski
  \[ SU(4) \times SU(2)_L \times SU(2)_R, SU(5) \subset SO(10) \]
  (Annals Phys. 93, 193(1975))

- Simple models ruled out : proton decay constraints!!
  Minimal SU(5) \( \tau_p > 10^{33} \) years

Thus look for more refined models(SUSY GUTS!!)
Motivations: Supersymmetry

Supersymmetric GUT have a no. of nice features associated with them.

- Gauge coupling Unification

- Susy solves Hierarchy Problem:

  \[ \Pi_{hh}(0) + \Pi_{hh}(0) \text{ free of quadratic divergences if } \lambda_s = -\lambda_f^2 \]

- Susy predictions: \( m_t \sim 200\text{ GeV} \) and \( \sin^2 \theta_w \sim .233 \)

SM: No meeting at one point. \( M_{\text{GUT}} \sim 10^{15} \text{ GeV} \)

MSSM: Couplings meet at a point \( M_{\text{GUT}} \sim 2 \times 10^{16} \text{ GeV} \)
SO(10): Generic Features

SO(10) GUT’s have a number of remarkable features which make them leading contender for BSM Physics.

- Spinor representation $SO(2n)$: $2^{n-1}$ dimension $SO(10)-16$ dimensional
- Parity breaking: Left-Right symmetry ($SU(4) \times SU(2)_L \times SU(2)_R$)
- Natural seesaw connection between neutrino mass and GUT scale

Seesaw:

$$M_\nu \sim \frac{v^2_W}{M_{B-L}} \Rightarrow M_{B-L} \sim 10^{14\pm 1} \text{GeV} \approx M_X!!!!!!$$

- Gauge unification can be achieved with or without supersymmetry
- Susy $SO(10)$: $R_P = (-1)^{3(B-L)+2s} \subset U(1)_{B-L}$ Certain class of these models $R$ parity conserving
SO(10): Representations

Under Pati-Salam ($SU(4) \times SU(2)_L \times SU(2)_R$):

- **Matter Supermultiplets**

  \[ 16 = (4, 2, 1) + (\bar{4}, 1, 2) \]

  \[ 16 \otimes 16 = 10 \oplus 120 \oplus 126 \Rightarrow 16 \cdot 16 \cdot (10 + 120 + \bar{126}) \]

- **Higgs Supermultiplets**

  \[ 10_H = (1, 2, 2) + (6, 1, 1) \]

  \[ \bar{126}_H = (10, 1, 3) + (\bar{10}, 3, 1) + (15, 2, 2) + (6, 1, 1) \]

  \[ 120_H = (10, 1, 1) + (\bar{10}, 1, 1) + (15, 2, 2) + (6, 1, 3) \]

  \[ + (6, 3, 1) + (1, 2, 2) \]

  \[ 210_H = (15, 1, 1) + (1, 1, 1) + (15, 1, 3) + (15, 3, 1) \]

  \[ + (6, 2, 2) + (10, 2, 2) + (\bar{10}, 2, 2) \]
Role Of Different Rep's

Thus a complete viable model can be constructed by choosing different rep's.

- 16 can contain \((Q_L, u^c_L, d^c_L, L_L, e_L \oplus \nu^c_L)\)
- Charged fermion Masses\((16 \cdot 16 \cdot (10 + \bar{126} + 120))\)
  \((1,2,2) \subset 10, (15,2,2) \subset \bar{126}, (1,2,2),(15,2,2) \subset 120\)
  Only one not sufficient!!...give bad mass relations!
- \(\bar{126}:\) Type I + Type II(seesaw masses!!)

\[
M_{\nu_R} = \langle (10, 1, 3) \rangle Y_{\bar{126}}; \quad M_{\nu_L} = \langle (\bar{10}, 3, 1) \rangle Y_{\bar{126}}
\]
\[
M_\nu = -M_{\nu_D} M_{\nu_R}^{-1} M_{\nu_D} + M_{\nu_L}
\]

\((15,1,1),(1,1,1),(15,1,3) \subset 210\)
Compltes symmetry breakdown: \(\text{SO}(10) \rightarrow \text{MSSM by 210.}\)
Minimal Supersymmetric Grand Unified Theory (MSGUT)

This theory was proposed long ago [1,2] but recently studied in detail.

- **Superpotential:** \((10 \oplus \overline{126} \oplus 126 \oplus 210)\)

\[
W_{AM} = \frac{1}{2} M_H H_i^2 + \frac{m}{4!} \Phi_{ijkl} \Phi_{ijkl} + \frac{\lambda}{4!} \Phi_{ijkl} \Phi_{klmn} \Phi_{mnij} + M \sum_{ijklm} \Phi_{ijkl} \sum_{ijklm} + \frac{\eta}{4!} \Phi_{ijkl} \sum_{ijmno} \sum_{klmno} + \frac{1}{4!} H_i \Phi_{jklm}(\gamma \sum_{ijklm} + \overline{\gamma} \sum_{ijklm})
\]

and

\[
W_{FM} = h_{AB} \psi_A^T C_2^{(5)} \gamma_i \psi_B H_i + \frac{1}{5!} f_{AB} \psi_A^T C_2^{(5)} \gamma_{i1} \ldots \gamma_{i5} \psi_B \overline{\sum_{i1} \ldots i5}
\]

- **26 Hard Parameters** (Minimal theory!!..Aulakh et al..hep-ph/0306242)

MSGUT: Symmetry breaking

\[ \langle (15, 1, 1) \rangle_{210} : \frac{a}{2} \]  
\[ \langle (15, 1, 3) \rangle_{210} : \omega \]  
\[ \langle (1, 1, 1) \rangle_{210} : p \]  
\[ \langle (10, 1, 3) \rangle_{126} : \bar{\sigma} \]  
\[ \langle (10, 1, 3) \rangle_{126} : \sigma. \]

The standard model vacuum in units of \((m/\lambda)\) are \(\tilde{\omega} = -x\) and

\[
\tilde{a} = \frac{(x^2 + 2x - 1)}{(1 - x)} ; \quad \tilde{p} = \frac{x(5x^2 - 1)}{(1 - x)^2} ; \quad \tilde{\sigma}\tilde{\sigma} = \frac{2 \lambda x(1 - 3x)(1 + x^2)}{\eta (1 - x)^2}
\]

where \(x\) is a solution of the cubic equation:

\[ 8x^3 - 15x^2 + 14x - 3 = -\xi (1 - x)^2 \]

with \(\xi = \frac{\lambda M}{\eta m}\). (see Aulakh etal refrences!!...)
Hall defines the matching functions $\lambda_i$ in terms of gauge couplings $\alpha_i$ to the grand unified coupling $\alpha_G$:

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_G(M_X)} + 8\pi b_i \ln \frac{M_X}{M_Z} + 4\pi \sum_j \frac{b_{ij}}{b_j} \ln X_j - 4\pi \lambda_i(M_X)$$

(9)

Here

$$X_j = 1 + 8\pi b_j \alpha_G(M_X^0) \ln \frac{M_X^0}{M_Z}$$

(10)

is understood to be evaluated at the values of $M_X^0, \alpha_G(M_X^0)$ determined from the one loop calculations.

$$\lambda_i(\mu) = -\frac{2}{21}(b_{iV} + b_{iGB}) + 2(b_{iV} + b_{iGB}) \ln \frac{M_V}{\mu}$$

(11)

$$+ 2b_{iS} \ln \frac{M_S}{\mu} + 2b_{iF} \ln \frac{M_F}{\mu}$$

(12)
One loop values $\alpha_G(M_X)$, $\sin^2\theta_w$ and $M_X$ remains stable against superheavy threshold corrections!![1]

$\sin^2\theta_w(M_Z)$ precisely known (0.1%) so usual to choose to predict $\alpha_3(M_Z)$ which carries largest uncertainty

$$
\Delta^{(th)}(\ln M_X) = \frac{\lambda_1(M_X) - \lambda_2(M_X)}{2(b_1 - b_2)}
$$

$$
\Delta^{(th)}(\alpha_3(M_Z)) = \frac{100\pi(b_1 - b_2)\alpha(M_Z)^2}{[(5b_1 + 3b_2 - 8b_3)\sin^2\theta_w(M_Z) - 3(b_2 - b_3)]^2}
\sum_{ijk} \epsilon_{ijk} (b_i - b_j) \lambda_k(M_X)
$$

$$
\Delta^{(th)}(\alpha_G^{-1}(M_X)) = \frac{4\pi(b_1\lambda_2(M_X) - b_2\lambda_1(M_X))}{b_1 - b_2}
$$

\[ \alpha_s(M_Z) = 0.130 \pm 0.001 + H_{\alpha_s} + \Delta_{\alpha_s} \quad (14) \]
\[ + 3.1 \times 10^{-7} \text{GeV}^{-2} \times [(m_t^{pole})^2 - (172.7 \text{GeV})^2] \quad (15) \]

\[ \Delta_{\alpha_s}^{\text{susy}} \approx -\frac{19\alpha_s^2}{28\pi} \ln \frac{M_{\text{susy}}}{M_Z} \quad \text{and} \quad -0.003 < H_{\alpha_s}(h_t, h_b) < 0. \]

For \( 250 \text{GeV} > M_{\text{SUSY}} > 20 \text{GeV} \) we get \( 0.005 > \Delta_{\alpha_s}^{\text{Susy}} > -0.003. \)

Thus \( -0.006 > \Delta_{\alpha_s}^{\text{GUT}} > -0.017 \) are required to reconcile with the measured value \( \alpha_3(M_Z) = 0.1176 \pm 0.002. \)

\[ |\Delta_G| \equiv |\Delta(\alpha^{-1}_G(M_X))| \leq 10 \]
\[ \Delta_X \equiv \Delta(\log_{10} M_X) \geq -1 \]
\[ -0.017 < \Delta_3 \equiv \Delta \alpha_3(M_Z) < -0.006 \quad (16) \]
Contour Plots of $\Delta \alpha_3(M_Z)$ on complex $x$ plane

- complex parameter $x$ is crucial parameter!!
- $\lambda, \eta, \gamma, \bar{\gamma} \sim 1$. 
MSGUT: Fermion mass formulae

Generic mass formulae in MSGUT:

\[
\begin{align*}
m^u &= v(\hat{h} + \hat{f}) \\
m^\nu &= v(\hat{h} - 3\hat{f}) \\
m^d &= v(r_1 \hat{h} + r_2 \hat{f}) \\
m^l &= v(r_1 \hat{h} - 3r_2 \hat{f})
\end{align*}
\] (17)

Here \( v = 174 \text{ GeV} \)

\[
\begin{align*}
M_{I\nu}^I &= vr_4 \hat{n} \\
M_{II\nu}^{II} &= 2vr_3 \hat{f}
\end{align*}
\] (18)

where \( \hat{n} = (\hat{h} - 3\hat{f})\hat{f}^{-1}(\hat{h} - 3\hat{f}) \)
In generic fits the required relative strength of Type I and Type II is simply assumed and there is no restriction of choosing the values of $r_1, r_2, r_3, r_4$

- Data for GUT To Explain: $m_{q,l}, \theta_i^{CKM}, \delta^{CKM}, M_\nu, \theta_i^{PMNS}$

- Babu and Mohapatra (1992): $\mathbf{10} \oplus \mathbf{126} \Leftrightarrow m_{q,l}$ Predictive in the Neutrino Sector! : failure (1992)


* Matsuda, Koide, Fukuyama, Nishiura (2002): Successful Type I, large $\theta^{PMNS}$ fit!
Bajc, Senjanovic, Vissani (2002)

\[ M_{\nu}^{II} \sim f < \Delta_L > \sim (M_d - M_l) \sim m_\tau \left( \frac{\epsilon^2}{\epsilon^2} \frac{(m_b - m_\tau)}{m_\tau} \right) \Rightarrow \]

MSSM : \( m_b = m_\tau (M_X) \) BSV : Large PMNS mixing Natural IF

\[ \frac{(m_b - m_\tau)}{m_\tau} \sim \epsilon^2 \]

Goh Mohapatra Ng : Type II : 3 generations , Real/Complex : Good Fits except \( \delta^{CKM} > \frac{\Pi}{2} \).

Bertolini, Malinsky (2004)(Type II, \( \oplus 120 \) ; Babu Macesanu (2005)(Type I and II) \textbf{Good Angle and Ratio Fits.} !!

Magnitude \( M_\nu \) and Relative Strength Type I vs Type II INPUT
MSGUT: Seesaw Mechanism

In fully specified model strength of Type I vs Type II and parameters $r_1, r_2, r_3, r_4$ are fixed in terms of GUT parameters!!

\[
\begin{align*}
    m^u &= \nu(\hat{h} + \hat{f}) \\
    m^\nu &= \nu(\hat{h} - 3\hat{f}) \\
    m^d &= \nu(r_1\hat{h} + r_2\hat{f}) \\
    m^l &= \nu(r_1\hat{h} - 3r_2\hat{f})
\end{align*}
\] (19)

Here $\nu = 174\,\text{GeV}$

\[
\begin{align*}
    \hat{h} &= 2\sqrt{2}h\alpha_1 \sin \beta; \quad \hat{f} = -4\sqrt{\frac{2}{3}}i\alpha_2 \sin \beta \\
    r_1 &= \frac{\bar{\alpha}_1}{\alpha_1} \cot \beta; \quad r_2 = \frac{\bar{\alpha}_2}{\alpha_2} \cot \beta
\end{align*}
\] (20)

where $\alpha_i, \bar{\alpha}_i$ are components of the null eigenvectors of the doublet mass matrix. superpotential.
MSGUT: Seesaw Mechanism

The fermion mass formulae in terms of GUT parameters are given by

\[
M_{\nu}^I = (1.70 \times 10^{-3} \text{eV}) \sin \beta F_I \hat{n}
\]

\[
M_{\nu}^{II} = (1.70 \times 10^{-3} \text{eV}) \sin \beta F_{II} \hat{f}
\]

where

\[
F_I = \frac{10^{-\Delta x}}{2\sqrt{2}} \frac{\gamma g}{\sqrt{\lambda \eta}} \left| p_2 p_3 p_5 \right| \sqrt{\frac{z_2}{z_{16}}} \sqrt{\frac{(1 - 3x)}{x(1 + x^2)}} \frac{q_3'}{p_5}
\]

\[
F_{II} = 10^{-\Delta x} \frac{2\gamma g}{\sqrt{\eta \lambda}} \frac{\left| p_2 p_3 p_5 \right|}{(x - 1)} \sqrt{\frac{z_2}{z_{16}}} \sqrt{\frac{(x^2 + 1)}{x(1 - 3x)}} \frac{(4x - 1)q_3^2}{q_3' q_2 p_5}
\]

\[
R = \left| \frac{F_I}{F_{II}} \right| = \left| \frac{(x - 1)(3x - 1)q_2 q_3'^2}{4\sqrt{2}(4x - 1)(x^2 + 1)q_3^2} \right|
\]
Type II seesaw: No dominance

Figure: $R^{-1}$ vs $x$ on complex plane.

- $|x| \to \infty$ it grows as $(3/(64\sqrt{2}))|x|$. For $|x| \geq 3 \Rightarrow R \geq 10^{-1}$ or so.

- This is in fact the region which contains the zeros of $R$ namely $x = \{1/3, 1, (3 \pm i\sqrt{7})/8, 0.198437, -0.0992186 \pm 2.24266i\}$ of which the last two sets are the zeros of $q_2$ and $q_3'$ respectively.
Type II seesaw: No dominance

\[ \{x_i, \epsilon_i, \bar{R}_{\text{min}}(x_i), \{\text{USMPs}\}\} \]
\[ = \{1, .014, .01, \{\Delta X < -9.79, \Delta \alpha_s < -0.433\}\} \]
\[ = \{.3333, .0025, .02, \{\Delta X > 4.86, \Delta \alpha_s > 0.063\}\} \]
\[ = \{.198437, .0025, 7 \times 10^{-5}, \{\Delta X < -3.2, \Delta \alpha_s < -0.019\}\} \]
\[ = \{-0.099219 + 2.2426i, .02, 2.4 \times 10^{-5}, \]
\[ \Delta \alpha_G^{-1} > 15.4, \Delta \alpha_s < -0.034\} \]

\[ \{x, \epsilon, \bar{R}_{\text{min}}(x_i), \{\text{USMPs}\}\} = ((3 + i\sqrt{7})/8, .06, .021, \{\Delta \alpha_s > .012, \Delta \alpha_G^{-1} > 18.8\} \)
\[ \langle 3 + i\sqrt{7}/8, .005, .002, \{\Delta \alpha_s > .059, \Delta \alpha_G^{-1} > 18.9\} \} \] (23)

The parameters of the MSGUT can never be chosen to ensure Type II domination while maintaining viable USMPs.
Type I: Not Strong enough

\[ M_\nu^I = 1.7 \times 10^{-3} eV F_I \hat{n} \sin \beta \]

\[ F_I = \frac{\gamma g}{2\sqrt{2}\eta \lambda} \sqrt{\frac{(1 - 3x)}{x(x^2 + 1)}} \frac{|p_2 p_3|\sqrt{Z_2}}{\sqrt{Z_{16}}} 10^{-\Delta x} q'_3 = \hat{F}_I 10^{-\Delta x} \]

(24)

\[ m_\nu \sim .05 eV \hat{F}_I \sim 10 (\hat{n} \sim 0.3 \text{ and } \Delta x > -1) \]

\[ |x| \to \infty \Rightarrow |\hat{F}_I| \to \sim .02 \]

\[ |x| \to \{0, \pm i\} \Rightarrow |\hat{F}_I| \to \infty \]

(25)

Figure: \(|\hat{F}_I|\) vs \(x\) on complex plane.
Type I: Not Strong enough

Around \( x=0, \Delta_X < -1 \), and near \( x \pm i, \Delta_X > 1 \) and hence suppress the favourable value of \( \hat{F}_I \).

Thus even with Type I seesaw there is no possibility of achieving neutrino masses larger than about \( 5 \times 10^{-3} \text{eV} \) in the MSGUT!
MSGUT: Conclusions

- Generically no type II dominance. Even near exceptional points no viable unification.
- Type I seesaw: $\left(m_\nu\right)_{max} \leq 5 \times 10^{-3}$ eV.
- Thus combined constraints of seesaw fit and stability of one loop gauge unification are enough to ruin the compatibility of the MSGUT with generic Type I and Type II seesaw mechanisms.
- Thus one should investigate the role of missing piece (i.e. $120$-plet).
New Minimal SUSY GUT

- NO SM VEV: Symmetry Pattern remains same

\[ O_{ijk}(120) = O^{(s)}_{\mu \nu}(10, 1, 1) + \bar{O}^{(s)}_{\mu \nu}(10, 1, 1) + O_{\nu \alpha \bar{\alpha}}^{\mu}(15, 2, 2) + O^{(a)}_{\mu \nu \alpha \bar{\beta}}(6, 1, 3) + O^{(a)}_{\mu \nu \alpha \bar{\beta}}(6, 3, 1) + O_{\alpha \bar{\alpha}}(1, 2, 2) \] (26)

- Superpotential: \( W_{NMSGUT} \)

\[
W_{NMSGUT} = \frac{m_o}{2(3!)} O_{ijk} O_{ijk} + \frac{k}{3!} O_{ijk} H_m \Phi_{mijk} + \frac{\rho}{4!} O_{ijk} O_{mnk} \Phi_{ijmn} \\
+ \frac{1}{2(3!)} O_{ijk} \Phi_{klmn}(\zeta \Sigma_{lmnij} + \bar{\zeta} \bar{\Sigma}_{lmnij}) \\
+ \frac{1}{5!} g_{AB} \psi^T_A C_2^{(5)} \gamma_{i_1} \gamma_{i_2} \gamma_{i_3} \psi_B O_{i_1i_2i_3} + W_{MSGUT}
\]
NMSGUT: RG Analysis

- Plot NMSGUT Allowed Region over complex $x$ plane.

![Figure: NMSGUT Allowed Region Over Complex $x$ plane.]

- Non diagonal couplings ($\gamma, \bar{\gamma}, \zeta, \bar{\zeta}, k$) minor influence on unification parameters diagonal ($\lambda, \eta, \rho$) have mild dependance except when coherent.

![Figure: NMSGUT Allowed Region Over Complex $x$ plane.]
Fermion mass relations

\[ m^u = v(\hat{h} + \hat{f} + \hat{g}) \quad ; \quad r_1 = \frac{\bar{\alpha}_1}{\alpha_1} \cot \beta \quad ; \quad r_2 = \frac{\bar{\alpha}_2}{\alpha_2} \cot \beta \]

\[ m^\nu = v(\hat{h} - 3\hat{f} + (r_5 - 3)\hat{g}) \quad ; \quad r_5 = \frac{4i\sqrt{3}\alpha_5}{\alpha_6 + i\sqrt{3}\alpha_5} \]

\[ m^d = v(r_1\hat{h} + r_2\hat{f} + r_6\hat{g}) \quad ; \quad r_6 = \frac{\bar{\alpha}_6 + i\sqrt{3}\bar{\alpha}_5}{\alpha_6 + i\sqrt{3}\alpha_5} \cot \beta \]

\[ m^l = v(r_1\hat{h} - 3r_2\hat{f} + (\bar{r}_5 - 3r_6)\hat{g}) \quad ; \quad \bar{r}_5 = \frac{4i\sqrt{3}\bar{\alpha}_5}{\alpha_6 + i\sqrt{3}\alpha_5} \cot \beta \]

\[ \hat{g} = 2ig\sqrt{\frac{2}{3}}(\alpha_6 + i\sqrt{3}\alpha_5) \sin \beta \quad ; \quad \hat{h} = 2\sqrt{2}h\alpha_1 \sin \beta \quad ; \quad (27) \]

\[ \hat{f} = -4\sqrt{\frac{2}{3}} if \alpha_2 \sin \beta \quad ; \quad (28) \]
NMSGUT Parameter Space

- Superpotential Parameters: $m, M, M_O, \lambda, \eta, \rho, k, \gamma, \tilde{\gamma}, \zeta, \tilde{\zeta}$.  
  21-4 (fixed by phase freedom from 5 Higgs fields) = 17  
  $M_H$ can be fixed by fine tuning to keep two pairs of doublets light.  
  Mass parameter $m$ is fixed by RG flow via fixing $M_X$.

$$|m| = 10^{16.25+\Delta x} \frac{|\lambda|}{g_5 \sqrt{2|\tilde{a} + \tilde{w}|^2 + |\tilde{p} + \tilde{\omega}|^2}} \text{GeV} = f(x, \lambda, \eta, \gamma...)$$

- Yukawa sector: 3 (h) + 12(f) + 6(g) = 21 parameters.
- Thus in full NMSGUT total 37 parameters are available for fitting of which 15 are phases.
- Data for GUT To Explain: $m_{q,l}, \theta_i^{CKM}, \delta^{CKM}, M_\nu, \theta_i^{PMNS}$
Fermion Fitting-NMSGUT-I

- Downhill Simplex:

\[ \chi^2 = \sum_i \left( \frac{f_i(x) - \bar{f}_i}{\delta f_i} \right)^2 \]

- This method does the function evaluations with reflection, contraction, extrapolation etc. to find the best possible minimum.
Solution obtained with $\chi^2 \sim 2.7$

The main features of solutions obtained are:

- Neutrino masses and mixing accurately fit
- $M_R \sim 10^8 - 10^{13} \text{Gev}$ ($|f| \sim 10^{-6}$)
- $y_d, y_s$ 2-3 standard deviations below the expected central values.

Thus the message is clear: unless the SUSY threshold corrections effects lower the $y_{d,s}$ the tree level NMSGUT formulae fail to fit the fermion data.
Susy Threshold Corrections

After matching SM with MSSM at SUSY scale yukawa couplings are related as[1,2]

\[ h_i^{MSSM} = \frac{h_i^{SM}}{\cos \beta (1 + \eta_i)} \]

\( \eta_i \) govern susy threshold corrections
\( \text{trig} \beta = \sin \beta \) for \( T_{3L} = +1/2 \) and \( \cos \beta \) for \( T_{3L} = -1/2 \).
So these corrections can give significant corrections to yukawa couplings in large \( \tan \beta \) limit.

Inclusion of Susy threshold corrections

- Fitting flow Chart

\[ M_1 = M_2 = M_3 = r_1 \]
\[ M_{H_1}^2 = r_2, \quad M_{H_2}^2 = r_3 \]
\[ M_f^2 = \text{Diagonal}(r_4, r_4, r_4) \]
\[ (A_e, A_d, A_u) = \text{Diagonal}(r_5, r_5, r_5) \otimes (Y_e, Y_d, Y_u) \]
\[ \chi^2 = \sum_i (1 - \frac{y_{i}^{\text{MSSM}}}{y_{i}^{\text{SM}}})^2 \]
\[ y_{i}^{\text{MSSM}} = y_{i}^{\text{MSSM}}(\text{trig} \beta (1 + \eta_i)) \]

\[ |\mu|^2 = \frac{1}{2} \left[ -M_Z^2 + \tan2\beta \left( -(M_{H_1}^2 - \frac{t_1}{v_1})\cot\beta + (M_{H_2}^2 - \frac{t_2}{v_2})\tan\beta \right) \right] \]
\[ B = \frac{1}{2} \left[ -M_Z^2 \sin2\beta - \tan2\beta \left( M_{H_1}^2 - \frac{t_1}{v_1} - M_{H_2}^2 + \frac{t_2}{v_2} \right) \right] \]
NMSGUT: Conclusions

- The model accurately fit the fermion data.
- only 5 GUT scale soft parameters!!
  (correcting mismatch between SM and MSSM yukawa couplings)
- 3rd generation sfermions are heavy then 1st and 2nd generation (a completely distinct signature of model!!)
- However one should test different yukawa fermion fits so that definite conclusions about soft spectra can be made.
- once it is done one can make predictions about proton decay, B physics etc..for this model
down quarks suffer corrections from squark/gluino($\tilde{s}\tilde{g}$)

$$\left\{ \frac{\Delta m_q}{m_q} \right\} \tilde{s}\tilde{g} = -\frac{g_3^2}{12\pi^2} \left\{ B_1(m_{\tilde{g}}, m_{\tilde{q}_1}; M_Z^2) + B_1(m_{\tilde{g}}, m_{\tilde{q}_2}; M_Z^2) ight. \\
\left. - \sin(2\theta_q) \left( \frac{m_{\tilde{g}}}{m_q} \right) [B_0(m_{\tilde{g}}, m_{\tilde{q}_1}; M_Z^2) - B_0(m_{\tilde{g}}, m_{\tilde{q}_2}; M_Z^2)] \right\}$$

$$\left\{ \frac{\Delta m_q}{m_q} \right\} \tilde{s}\tilde{\chi}^+ = -\frac{y_q^2}{16\pi^2} \mu a \frac{A_0 \tan\beta - mu}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2} \left\{ B_1(m_{\tilde{g}}, m_{\tilde{q}_1}; M_Z^2) ight. \\
\left. + B_1(m_{\tilde{g}}, m_{\tilde{q}_2}; M_Z^2) - \sin(2\theta_q) \left( \frac{m_{\tilde{g}}}{m_q} \right) [B_0(m_{\tilde{g}}, m_{\tilde{q}_1}; M_Z^2) \\
- B_0(m_{\tilde{g}}, m_{\tilde{q}_2}; M_Z^2)] \right\}$$

Leptons corrections small colorless particles

$$\left( \frac{\Delta m_l}{m_l} \right) = -\frac{g^2}{16\pi^2} \mu M_2 \tan\beta \left\{ B_0(M_2, m_{\nu_l}; M_Z^2) - B_0(M_2, m_{\nu_l}; M_Z^2) \right\}$$
Susy Threshold Corrections

- for up quarks no $\tan\beta$ enhanced corrections
  squark-gluino contribution is much similar to down quarks
- top quark-gluon correction are given by

\[
\left( \frac{\Delta m_t}{m_t} \right)^{tg} = \frac{g_3^2}{6\pi^2} \left[ 2B_0[m_t, m_t; M_Z^2] - B_1[m_t, m_t; M_Z^2] \right]
\]

Here $B_0(m_1, m_2; Q^2) = -\ln\left(\frac{M^2}{Q^2}\right) + 1 + \frac{m^4}{m^2 - M^2} \ln\left(\frac{M^2}{m^2}\right)$

\[
B_0(m_1, m_2; Q^2) = \frac{1}{2} \left[ \frac{M^2}{Q^2} + \frac{1}{2} + \frac{1}{1-x} + \frac{\ln x}{(1-x)^2} - \theta(1-x)\ln x \right]
\]

$M = \max(m_1, m_2)$, $m = \min(m_1, m_2)$ and $x = m_2^2/m_1^2$ and $\theta$ is unit step function

- for chargino corrections
  $\eta_i = \epsilon_i \tan\beta$ so with $\epsilon_i \sim 10^{-2} - 10^{-1}$ and $\tan\beta \sim 50 \eta_i \sim 0.5 - 5$