Some versions of the 3-3-1 model

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Outline

1. Introduction

There are four fundamental interactions of nature: gravitation, electromagnetism, weak interaction, and strong interaction with force carriers gravitons, photons, gauge bosons, gluons, correspondingly.

The electromagnetic, strong, and weak interactions associate with elementary particles.

- Weak interactions before gauge theory: One introduced V-A theory of four fermions and the weak current has form $J^\mu_V(vector) - J^\mu_A(axial)$. The theory is not renormalizable theory.

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- Some successful gauge theories of particle physics:
  - Q.E.D (Electromagnetism): $U(1)_{e.m}$
  - Weigberg/Salam (Electroweak): $SU(2)_L \otimes U(1)_Y$
  - Q.C.D (Strong): $SU(3)_C$

  $\rightarrow$ Standard Model of strong and Electroweak interactions based on the gauge groups $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.

Rules for coupling construction

- Lorentz invariance
- Charge conservation
- Renormalization

Review Standard Model

Fermion content of the SM

$$\begin{pmatrix} \nu_{aL} \\ e_{aL} \end{pmatrix} \sim (1,2,-1), \quad \begin{pmatrix} u_{aL} \\ d_{aL} \end{pmatrix} \sim (3,2,1/3), \quad a = 1,2,3$$
$$e_{aR} \sim (1,1,-2), \quad u_{aR} \sim (3,1,4/3), \quad d_{aR} \sim (3,1,-2/3).$$

The electric charge operator is defined as

$$Q = T_3 + \frac{1}{2}Y,$$

where $T_i = \frac{1}{2}\sigma_i \quad (i = 1,2,3)$, with σ_i are Pauli matrices. Y is hypercharge.

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- Before symmetry breaking: There are four massless gauge bosons. Higgs field with four components.

- After symmetry breaking: There is one massless photon γ and three massive gauge bosons $W^{\pm}, Z^0$. One leftover massive Higgs boson, three Goldstone bosons.

Problems in SM

- ▷ In SM neutrinos are massless. However, experiments suggest neutrinos masses.
- ▷ Generation problem:
  The number of generations is computed from the invisible width of the Z^0 within the framework of the standard model, then $N_{\text{gen}} = N_\nu = \Gamma_{\text{inv}}/\Gamma_\nu = 2.99 \pm 0.03$ but we do not understand why the number of standard model generations is three.

The general anomaly-free condition is

$$A^{ijk} = \sum_{\text{representations}} \text{Tr}[\{T_L^i, T_L^j\}T_L^k - \{T_R^i, T_R^j\}T_R^k] = 0,$$

where $T^i_{L,R}$ are the coupling matrices of fermions $\psi_{L,R}$ to the current

$$J^i_\mu = \bar{\psi}_L \gamma_\mu T^i_L \psi_L + \bar{\psi}_R \gamma_\mu T^i_R \psi_R.$$

The i index runs over the dimension of a simple $SU(n)$ group, $i = 1, 2, ..., n^2 - 1$.

- $\text{Tr}[\{\sigma^i, \sigma^j\}\sigma^k] = 2\delta^{ij}\text{Tr}[\sigma^k] = 0,$
- $\text{Tr}[\sigma^i YY] \propto \text{Tr}[\sigma^i] = 0,$
- $\text{Tr}[\{\sigma^i, \sigma^j\}Y] = 2\delta^{ij}\text{Tr}[Y] \sim (3\times[(-1)+3\times(1/3)]-[-2+3\times(4/3)+3\times(-2/3)] = 0,$
- $\text{Tr}[Y^3] = (3\times[(-1)^3 + 3\times(1/3)^3] - [-2^3 + 3\times(4/3)^3 + 3\times(-2/3)^3] = 0.$

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The $\mathrm{Tr}[Y]$ vanishes for the fermion content in the a^{th}-generation.

$\Rightarrow$ The anomaly in SM cancels within each individual generation, but not by generations. Flavor question and anomaly-free conditions do not seem to have any connection in the SM.

Motivation of the 3-3-1 models

- ▷ The obvious experimental evidence proves that we must go beyond SM.
- ▷ Among the beyond SM extensions, the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, called 3-3-1 models can
  - ▷ give small neutrino mass
  - ▷ explain number of generation
  - ▷ explain that the third quark generation has to be different from the first two.

2.Some main versions of the 3-3-1 model

There are some versions of 3-3-1 model, it depends on the electric charge of particle at the bottom of the lepton triplet.

▷ The minimal 3-3-1 model

▷ The 3-3-1 model with right handed neutrinos → The economical 3-3-1 model

▷ The 3-3-1 model with neutral fermions

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- ▷ The 3-3-1 model with right handed neutrinos → The economical 3-3-1 model
- ▷ The 3-3-1 model with neutral fermions

The new charge X is connected with the electric charge operator Q through a relation

$$Q = \alpha T_3 + \beta T_8 + XI,$$

where $T_i = \frac{1}{2}\lambda_i (i = 1, 2, 3, ..., 8)$, λ_i are Gell-Mann matrices.
The values α, β, X depend on specific 3-3-1 model.

Lepton number

- In the SM, the lepton number operator commutes with the generators of the unitary group $SU(2)_L$.
- In the 3-3-1 model the lepton number of 3 components in a triplet are different, so the lepton number operator does not commute with the generators of the unitary group $SU(3)_L$.
  For example $\psi_{aL} = (\nu_{aL}, e_{aL}, (N_{aR})^c)^T$ *has the lepton number (1,1,0) then the commutations* $[L, T_4], [L, T_5], [L, T_6], [L, T_7] \neq 0$, *where* T_4, T_5, T_6, T_7 *are the generators of* $SU(3)_L$ *containing new gauge bosons* X, Y.

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- In the 3-3-1 model, one constructed lepton number operator as the combination of T_3, T_8, and charged $\mathcal{L}$. One considered $U(1)_{\mathcal{L}}$ as global group.
  The relation between L and $\mathcal{L}$ is obtained

$$L = \alpha' T_3 + \beta' T_8 + \mathcal{L} I.$$

Fermion triplets in 3-3-1 model

In general, fermion triplets in 3-3-1 model are arranged as

$$\begin{aligned} \psi_{aL} &= \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ F_{aL} \end{pmatrix} \sim (1, 3, X_\psi), \\ Q_{\alpha L} &= \begin{pmatrix} d_{\alpha L} \\ -u_{\alpha L} \\ D_{\alpha L} \end{pmatrix} \sim (3, 3^*, X_{Q_\alpha}), \\ Q_{3L} &= \begin{pmatrix} u_{3L} \\ d_{3L} \\ U_L \end{pmatrix} \sim (3, 3, X_{Q_3}), \end{aligned}$$

where the quantum numbers located in the parentheses are defined upon the gauge symmetries $(SU(3)_C,\ SU(3)_L,\ U(1)_X)$, respectively. The family indices are $a = 1, 2, 3$ and $\alpha = 1, 2$.

The F_{aL} are the leptons corresponding to specific model and U, D_α are the exotic quarks.

The minimal 3-3-1 model

$$\psi_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (e_{aR})^c \end{pmatrix}, \quad (e_{aR})^c \equiv (e^c)_{aL}.$$

$$Q\psi_{aL} = (0, -1, 1) \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (e_{aR})^c \end{pmatrix} \Rightarrow \alpha = 1, \beta = -\sqrt{3}.$$

The operator $Q = T_3 - \sqrt{3}T_8 + XI$ and we get

$$X_\psi = 0, \quad X_{Q_\alpha} = -1/3, \quad X_{Q_3} = 2/3, \quad q_U = 5/3, \quad q_{D_\alpha} = -4/3.$$

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$$L\psi_{aL} = (1, 1, -1) \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (e_{aR})^c \end{pmatrix} \Rightarrow \alpha' = 0, \beta' = \frac{4}{\sqrt{3}}, \quad L = \frac{4}{\sqrt{3}}T_8 + \mathcal{L}I.$$

$L(D_\alpha) = -L(U) = L(Y^-, X^0) = 2.$

The 3-3-1 model with right handed neutrinos

$$\psi_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (\nu_{aR})^c \end{pmatrix}, \quad (\nu_{aR})^c \equiv (\nu^c)_{aL}.$$

$$Q\psi_{aL} = (0, -1, 0) \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (\nu_{aR})^c \end{pmatrix} \Rightarrow \alpha = 1, \beta = -\frac{1}{\sqrt{3}}.$$

The operator $Q = T_3 - \frac{1}{\sqrt{3}} T_8 + XI$ and we get

$$X_\psi = -1/3, \quad X_{Q_\alpha} = 0, \quad X_{Q_3} = 1/3, \quad q_U = 2/3, \quad q_{D_\alpha} = -1/3.$$

Electric charges of the exotic quarks are the same as of the usual quarks.
Lepton number operator has the same form as in the minimal model.

The 3-3-1 model with neutral fermions

$$\psi_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (N_{aR})^c \end{pmatrix}, \quad (N_{aR})^c \equiv (N^c)_{aL}.$$

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The operator Q and therefore $X_\psi, X_{Q_\alpha}, X_{Q_3}, q_U, q_{D_\alpha}$ are the same as in the model with right handed neutrinos.

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$$L\psi_{aL} = (1, 1, 0) \begin{pmatrix} \nu_{aL} \\ N_{aL} \\ (e_{aR})^c \end{pmatrix} \Rightarrow \alpha' = 0, \beta' = \frac{2}{\sqrt{3}}, \quad L = \frac{2}{\sqrt{3}} T_8 + \mathcal{L} I.$$

$L(D_\alpha) = -L(U) = L(Y^-, X^0) = 1.$

$$
\begin{aligned}
\psi_{aL} &\sim (1,3,-1/3), \quad Q_{\alpha L} \sim (3,3^*,0), \quad Q_{3L} \sim (3,3,1/3)\,, \\
\nu_{aR} &\sim (1,1,0), \quad e_{aR} \sim (1,1,-1), \\
u_{aR} &\sim (3,1,2/3)\,, \quad d_{aR} \sim (3,1,-1/3)\,, \\
U_R &\sim (3,1,2/3)\,, \quad D_{\alpha R} \sim (3,1,-1/3)\,,
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In 3-3-1 model there are six triangle anomalies which are potentially troublesome.

$$[SU(3)_C]^3, [SU(3)_C]^2[U(1)_X], [SU(3)_L]^3, [SU(3)_L]^2[U(1)_X], [U(1)_X]^3, [\text{graviton}]^2[U(1)_X].$$

The anomaly $[SU(3)_C]^3$ is absent because the theory is vector-like.

The anomaly $[SU(3)_L]^3$ vanishes because there is an equal number of triplets 3_L and antitriplets 3^*_L in the given fermion content.

The remaining anomaly-free conditions are explicitly written as follows

1. $\mathrm{Tr}[SU(3)_C]^2[U(1)_X] = 3\sum\limits_{\text{generation}} X_q^L - \sum\limits_{\text{generation}}\sum\limits_{\text{singlet}} X_q^R = 0,$
2. $\mathrm{Tr}[SU(3)_L]^2[U(1)_X] = \sum\limits_{\text{generation}} X_l^L + 3\sum\limits_{\text{generation}} X_q^L = 0,$
3. $\mathrm{Tr}[U(1)_X]^3 = 3\sum\limits_{\text{generation}} (X_l^L)^3 + 9\sum\limits_{\text{generation}} (X_q^L)^3$
   $-3\sum\limits_{\text{generation}}\sum\limits_{\text{singlet}} (X_q^R)^3 - \sum\limits_{\text{generation}}\sum\limits_{\text{singlet}} (X_l^R)^3 = 0,$
4. $\mathrm{Tr}[\text{graviton}]^2[U(1)_X] = 3\sum\limits_{\text{generation}} X_l^L + 9\sum\limits_{\text{generation}} X_q^L$
   $-3\sum\limits_{\text{generation}}\sum\limits_{\text{singlet}} X_q^R - \sum\limits_{\text{generation}}\sum\limits_{\text{singlet}} X_l^R = 0,$

where $X_l^L, X_q^L, X_l^R, X_q^R$ indicate to the $U(1)_X$ charges of the left-handed lepton, quark triplets or antitriplets and the right-handed lepton, quark singlets, respectively.
The factor 3 is the number of quark colors.

For the $[SU(3)_L]^2[U(1)_X]$:
The anomaly of first generation: $-1/3 + 3 \times (1/3) = 2/3$.
The anomaly of the second or the third generation: $-1/3 + 3 \times 0 = -1/3$.
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Each individual generation possesses non-vanishing anomalies

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In general,

$$-1/3 + N_q \times (1/3) + (N_\nu - 1) \times (-1/3 + N_q \times 0) = 0 \Rightarrow N_\nu = N_q.$$

$\Rightarrow$ Only with a matching of the number of generations with the number of quark colors does the overall anomaly vanish.

3. Extension of 3-3-1 model to 3-3-1-1 model

Motivation of 3-3-1-1 model with neutral fermions

- There exists a simple extension of the SM gauge group to $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$, the so called 3-3-1 models. There are three main versions: the minimal model, the version with right-handed (RH) neutrinos and the version with neutral fermions.

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- ▷ In 3-3-1 model, the lepton number L is constructed as the combination of T_3, T_8, and charged $\mathcal{L}$ and L is considered as global symmetry.

- ▷ Since T_3, T_8 are gauged charges of the $SU(3)_L$ symmetry, $L, \mathcal{L}$ should be gauged or local generators. In 3-3-1-1 model, the lepton number is considered as local symmetry.

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- ▷ The new particles in the version with neutral fermions have lepton number $\pm 1 \rightarrow$ They are odd under parity operator.

The 3-3-1-1 model is based on the gauge symmetry

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N.$$

$T_i = \frac{1}{2}\lambda_i (i = 1, 2, 3, ..., 8)$ and X, N are $SU(3)_L$, $U(1)_X$ and $U(1)_N$ charges, respectively. λ_i are Gell-Mann matrices.

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$$[SU(3)_C]^2 U(1)_{\mathcal{L}}, [SU(3)_L]^2 U(1)_{\mathcal{L}}, [U(1)_X]^2 U(1)_{\mathcal{L}}, U(1)_X [U(1)_{\mathcal{L}}]^2, [U(1)_{\mathcal{L}}]^3 \neq 0.$$

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- ▷ Define $B = \mathcal{B}I, N = \mathcal{B} - \mathcal{L}$, the anomalies associated with $U(1)_N$ and with the usual $3-3-1$ symmetry obviously vanish.

To break the gauge symmetry and generate the masses in a correct way, the 3-3-1-1 model needs the following scalar multiplets :

$$\rho = \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^+ \end{pmatrix} \sim (1,3,2/3,1/3),$$

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} \sim (1,3,-1/3,1/3),$$

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix} \sim (1,3,-1/3,-2/3),$$

$$\phi \sim (1,1,0,2),$$

with the VEVs conserving Q and P respectively given by

$$\langle\rho\rangle = \frac{1}{\sqrt{2}}(0,v,0)^T, \quad \langle\eta\rangle = \frac{1}{\sqrt{2}}(u,0,0)^T, \quad \langle\chi\rangle = \frac{1}{\sqrt{2}}(0,0,\omega)^T, \quad \langle\phi\rangle = \frac{1}{\sqrt{2}}\Lambda.$$

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The gauge group $SU(3)_L \otimes U(1)_X \otimes U(1)_N$ is broken:

$$SU(3)_L \otimes U(1)_X \otimes U(1)_N \to U(1)_Q \otimes U(1)_{B-L}.$$

The $\mathcal{L}, \mathcal{B}, N$ charge of model multiplets

Multiplet	ψ_{aL}	ν_{aR}	e_{aR}	Q_{3L}	$Q_{\alpha L}$	u_{aR}	d_{aR}	U_R	$D_{\alpha R}$	ρ	η	χ	ϕ
$\mathcal{L}$	$\frac{2}{3}$	1	1	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	-1	1	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	-2
$\mathcal{B}$	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0
$N = \mathcal{B} - \mathcal{L}$	$-\frac{2}{3}$	-1	-1	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	2

The lepton number of model particles

L	Particle
0	$N_{aR}, u_a, d_a, \rho_1^+, \rho_2^0, \eta_1^0, \eta_2^-, \chi_3^0$
1	$\nu_{aL}, e_a, \bar{U}, D_\alpha, \rho_3^-, \eta_3^{0*}, \chi_1^0, \chi_2^-$

The R parity of model particles

$P = (-1)^{3(B-L)+2s}$	Particle
+1	$\nu_L, \nu_R, e, u, d, \rho_1, \rho_2, \eta_1, \eta_2, \chi_3, \phi$
-1	$N_R, U, D, \rho_3, \eta_3, \chi_1, \chi_2$

The Lagrangian of the 3-3-1-1 model is given by

$$\begin{aligned}\mathcal{L} = & \sum_{\text{fermion multiplets}} \bar{F} i\gamma^\mu D_\mu F + \sum_{\text{scalar multiplets}} (D^\mu S)^\dagger (D_\mu S) \\ & -\frac{1}{4} G_{i\mu\nu} G_i^{\mu\nu} - \frac{1}{4} A_{i\mu\nu} A_i^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} C_{\mu\nu} C^{\mu\nu} \\ & -V(\rho, \eta, \chi, \phi) + \mathcal{L}_{\text{Yukawa}},\end{aligned}$$

with the covariant derivative

$$D_\mu = \partial_\mu + i g_s T_i G_{i\mu} + i g T_i A_{i\mu} + i g_X X B_\mu + i g_N N C_\mu,$$

and the field strength tensors

$$\begin{aligned}G_{i\mu\nu} &= \partial_\mu G_{i\nu} - \partial_\nu G_{i\mu} - g_s f_{ijk} G_{j\mu} G_{k\nu}, \\ A_{i\mu\nu} &= \partial_\mu A_{i\nu} - \partial_\nu A_{i\mu} - g f_{ijk} A_{j\mu} A_{k\nu}, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \quad C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu.\end{aligned}$$

The Yukawa interactions and scalar potential are obtained as

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= h^e_{ab}\bar{\psi}_{aL}\rho e_{bR} + h^{\nu}_{ab}\bar{\psi}_{aL}\eta\nu_{bR} + h'^{\nu}_{ab}\bar{\nu}^c_{aR}\nu_{bR}\phi + h^U\bar{Q}_{3L}\chi U_R + h^D_{\alpha\beta}\bar{Q}_{\alpha L}\chi^* D_{\beta R} \\ &+ h^u_a\bar{Q}_{3L}\eta u_{aR} + h^d_a\bar{Q}_{3L}\rho d_{aR} + h^d_{\alpha a}\bar{Q}_{\alpha L}\eta^* d_{aR} + h^u_{\alpha a}\bar{Q}_{\alpha L}\rho^* u_{aR} + H.c,\end{aligned}$$

$$\begin{aligned}V(\rho,\eta,\chi,\phi) &= \mu_1^2\rho^\dagger\rho + \mu_2^2\chi^\dagger\chi + \mu_3^2\eta^\dagger\eta + \lambda_1(\rho^\dagger\rho)^2 + \lambda_2(\chi^\dagger\chi)^2 + \lambda_3(\eta^\dagger\eta)^2 \\ &+ \lambda_4(\rho^\dagger\rho)(\chi^\dagger\chi) + \lambda_5(\rho^\dagger\rho)(\eta^\dagger\eta) + \lambda_6(\chi^\dagger\chi)(\eta^\dagger\eta) \\ &+ \lambda_7(\rho^\dagger\chi)(\chi^\dagger\rho) + \lambda_8(\rho^\dagger\eta)(\eta^\dagger\rho) + \lambda_9(\chi^\dagger\eta)(\eta^\dagger\chi) + (f\epsilon^{mnp}\eta_m\rho_n\chi_p + H.c.) \\ &+ \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 + \lambda_{10}(\phi^\dagger\phi)(\rho^\dagger\rho) + \lambda_{11}(\phi^\dagger\phi)(\chi^\dagger\chi) + \lambda_{12}(\phi^\dagger\phi)(\eta^\dagger\eta).\end{aligned}$$

Scalar sector

We expand the neutral scalars around their VEVs such as

$$\rho = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(v + S_2 + iA_2) \\ \rho_3^+ \end{pmatrix}; \eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(u + S_1 + iA_1) \\ \eta_2^- \\ \frac{1}{\sqrt{2}}(S_3' + iA_3') \end{pmatrix}; \chi = \begin{pmatrix} \frac{1}{\sqrt{2}}(S_1' + iA_1') \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(\omega + S_3 + iA_3) \end{pmatrix};$$

$$\phi \sim \frac{1}{\sqrt{2}}(\Lambda + S_4 + iA_4).$$

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$$\phi \sim \frac{1}{\sqrt{2}}(\Lambda + S_4 + iA_4).$$

We assume that f, ω are the same order and $\omega \gg u, v$. The physical fields with respective masses can be written as:
For charged scalars,

$$H_4^- = \frac{v\chi_2^- + \omega\rho_3^-}{\sqrt{v^2+\omega^2}}, \quad H_5^- = \frac{v\eta_2^- + u\rho_1^-}{\sqrt{u^2+v^2}},$$

$$m_{H_4}^2 = \left(\frac{1}{2}\lambda_7 - \frac{fu}{\sqrt{2}v\omega}\right)(v^2 + \omega^2), \quad m_{H_5}^2 = \left(\frac{1}{2}\lambda_8 - \frac{f\omega}{\sqrt{2}uv}\right)(u^2 + v^2).$$

$$G_Y^- = \frac{-\omega\chi_2^- + v\rho_3^-}{\sqrt{v^2+\omega^2}}, G_W^- = \frac{-u\eta_2^- + v\rho_1^-}{\sqrt{u^2+v^2}}.$$

The pseudoscalars,

$$m_{A_4} = 0, \quad m_{G_X} = 0, \quad m_{G_Z} = 0, \quad m_{G_{Z'}} = 0,$$

$$G_X = \frac{\omega\chi_1^0 - u\eta_3^{0*}}{\sqrt{u^2+\omega^2}}, \quad G_Z = \frac{uA_1 - vA_2}{\sqrt{u^2+v^2}},$$

$$G_{Z'} = \frac{-\omega^{-1}(u^{-1}A_1 + v^{-1}A_2) + (u^{-2}+v^{-2})A_3}{\sqrt{(u^{-2}+v^{-2}+\omega^{-2})(u^{-2}+v^{-2})}}.$$

$$A = \frac{u^{-1}A_1 + v^{-1}A_2 + \omega^{-1}A_3}{\sqrt{u^{-2}+v^{-2}+\omega^{-2}}}, \quad m_A^2 = -\frac{f}{\sqrt{2}}\frac{u^2v^2+u^2\omega^2+v^2\omega^2}{uv\omega}.$$

$$H' = \frac{u\chi_1^0 + \omega\eta_3^{0*}}{\sqrt{u^2+\omega^2}}, \quad m_{H'}^2 = (\frac{1}{2}\lambda_9 - \frac{fv}{\sqrt{2}u\omega})(u^2+\omega^2).$$

For neutral scalars, S_1, S_2, S_3, S_4, all mix via the mass Lagrangian,

$$\frac{1}{2}(S_1\ S_2\ S_3\ S_4)M_S^2(S_1\ S_2\ S_3\ S_4)^T,$$

where the squared-mass matrix is given by

$$M_S^2 = \begin{pmatrix} 2\lambda_3 u^2 - \frac{1}{\sqrt{2}}f\frac{v\omega}{u} & \lambda_5 uv + \frac{1}{\sqrt{2}}f\omega & \lambda_6 u\omega + \frac{1}{\sqrt{2}}fv & \lambda_{12}u\Lambda \\ \lambda_5 uv + \frac{1}{\sqrt{2}}f\omega & 2\lambda_1 v^2 - \frac{1}{\sqrt{2}}f\frac{u\omega}{v} & \lambda_4 \omega v + \frac{1}{\sqrt{2}}fu & \lambda_{10}v\Lambda \\ \lambda_6 u\omega + \frac{1}{\sqrt{2}}fv & \lambda_4 \omega v + \frac{1}{\sqrt{2}}fu & 2\lambda_2\omega^2 - \frac{1}{\sqrt{2}}f\frac{vu}{\omega} & \lambda_{11}\omega\Lambda \\ \lambda_{12}u\Lambda & \lambda_{10}v\Lambda & \lambda_{11}\omega\Lambda & 2\lambda\Lambda^2 \end{pmatrix}.$$

In the limit $\Lambda \sim \omega$

$$\begin{aligned}
H &= \frac{uS_1 + vS_2}{\sqrt{u^2+v^2}}, \quad m_H^2 = 2\left(\frac{\lambda_3 u^4 + \lambda_5 u^2 v^2 + \lambda_1 v^4}{u^2+v^2} + m_0^2 + m_1^2 \frac{f}{\omega} + m_2^2 \frac{f^2}{\omega^2}\right),\\
H_1 &= \frac{-vS_1 + uS_2}{\sqrt{u^2+v^2}}, \quad m_{H_1}^2 = -\frac{f(u^2+v^2)\omega}{\sqrt{2}uv},\\
H_2 &= \cos\varphi S_3 + \sin\varphi S_4, \quad m_{H_2}^2 = \lambda_2\omega^2 + \lambda\Lambda^2 - \sqrt{\lambda_2^2\omega^4 + (\lambda_{11}^2 - 2\lambda\lambda_2)\omega^2\Lambda^2 + \lambda^2\Lambda^4},\\
H_3 &= -\sin\varphi S_3 + \cos\varphi S_4, \quad m_{H_3}^2 = \lambda_2\omega^2 + \lambda\Lambda^2 + \sqrt{\lambda_2^2\omega^4 + (\lambda_{11}^2 - 2\lambda\lambda_2)\omega^2\Lambda^2 + \lambda^2\Lambda^4},
\end{aligned}$$

where $\tan 2\varphi \equiv -\frac{\lambda_{11}\omega\Lambda}{\lambda\Lambda^2 - \lambda_2\omega^2}$.

The mass parameters $m_0,\ m_1,\ m_2$ are given as follows

$$\begin{aligned}
m_0^2 &\equiv -\frac{1}{(\lambda_{11}^2 - 4\lambda\lambda_2)(v^2+u^2)}\left[-\lambda_{12}^2\lambda_2 u^4 - \lambda(\lambda_6 u^2 + \lambda_4 v^2)^2\right.\\
&\quad \left. + \lambda_{12}u^2(\lambda_{11}\lambda_6 u^2 - 2\lambda_{10}\lambda_2 v^2 + \lambda_{11}\lambda_4 v^2) + \lambda_{10}v^2(\lambda_{11}\lambda_6 u^2 - \lambda_{10}\lambda_2 v^2 + \lambda_{11}\lambda_4 v^2)\right],\\
m_1^2 &\equiv -\frac{\sqrt{2}uv\left[(\lambda_{11}\lambda_{12} - 2\lambda\lambda_6)u^2 + (\lambda_{10}\lambda_{11} - 2\lambda\lambda_4)v^2\right]}{(\lambda_{11}^2 - 4\lambda\lambda_2)(u^2+v^2)},\\
m_2^2 &\equiv \frac{2\lambda u^2 v^2}{(\lambda_{11}^2 - 4\lambda\lambda_2)(u^2+v^2)}.
\end{aligned}$$

In the limit $\Lambda \gg \omega$

$$
\begin{aligned}
H &= \frac{uS_1 + vS_2}{\sqrt{u^2+v^2}}, \quad m'^2_H = \frac{v^4(4\lambda\lambda_1 - \lambda_{10}^2) - u^4(\lambda_{12}^2 - 4\lambda\lambda_3) - 2u^2v^2(\lambda_{10}\lambda_{12} - 2\lambda\lambda_5)}{2(u^2+v^2)\lambda} \\
&\quad + \frac{1}{2\sqrt{2}(u^2+v^2)\lambda(\lambda_{11}^2 - 4\lambda\lambda_2)}(m_0 + m_1\frac{f}{\omega} + m_2\frac{f^2}{\omega^2}), \\
H_1 &= \frac{-vS_1 + uS_2}{\sqrt{u^2+v^2}}, \quad m^2_{H_1} = -\frac{f(u^2+v^2)\omega}{\sqrt{2}uv}, \\
H_2 &= S_3, \quad m^2_{H_2} = \frac{(4\lambda\lambda_2 - \lambda_{12}^2)\omega^2}{2\lambda}, \\
H_3 &\simeq S_4, \quad m^2_{H_3} = 2\lambda\Lambda^2,
\end{aligned}
$$

where

$$
\begin{aligned}
m_0^2 &\equiv \sqrt{2}(v^2(\lambda_{10}\lambda_{11} - 2\lambda\lambda_4) + u^2(\lambda_{11}\lambda_{12} - 2\lambda\lambda_6))^2, \\
m_1^2 &\equiv 8uv\lambda(v^2(-\lambda_{10}\lambda_{11} + 2\lambda\lambda_4) + u^2(-\lambda_{11}\lambda_{12} + 2\lambda\lambda_6)), \\
m_2^2 &\equiv 8\sqrt{2}u^2v^2\lambda^2.
\end{aligned}
$$

To have a numerical value for the Higgs mass, we consider $u = v,\ \omega = -f$. Therefore, the Higgs mass is given by

$$m_H^2 = \bar{\lambda} u^2,$$

where $\bar{\lambda}$ is the function of only the λ's couplings.

$$\bar{\lambda} \simeq 0.5 \to m_H = 125 \text{ GeV}.$$

H is identified as SM Higgs boson.

4. Gauge bosons

The gauge mass Lagrangian is given as

$$\begin{aligned}
\mathcal{L}_{gaugemass} &= (0,0,\frac{\omega}{\sqrt{2}})(gA_{a\mu}T_a - \frac{1}{3}g_X B_\mu - \frac{2}{3}g_N C_\mu)^2(0,0,\frac{\omega}{\sqrt{2}})^T \\
&+(\frac{u}{\sqrt{2}},0,0)(gA_{a\mu}T_a - \frac{1}{3}g_X B_\mu + \frac{1}{3}g_N C_\mu)^2(\frac{u}{\sqrt{2}},0,0)^T \\
&+(0,\frac{v}{\sqrt{2}},0)(gA_{a\mu}T_a + \frac{2}{3}g_X B_\mu + \frac{1}{3}g_N C_\mu)^2(0,\frac{v}{\sqrt{2}},0)^T \\
&+2(g_N C_\mu \Lambda)^2.
\end{aligned}$$

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$$W^\pm_\mu = \frac{A_{1\mu} \mp iA_{2\mu}}{\sqrt{2}}, Y^\mp_\mu = \frac{A_{6\mu} \mp iA_{7\mu}}{\sqrt{2}}.$$

$$M^2_W = \frac{1}{4}g^2(u^2+v^2), M^2_Y = \frac{1}{4}g^2(v^2+\omega^2).$$

$$X^0_\mu = \frac{A_{4\mu} - iA_{5\mu}}{\sqrt{2}}, M^2_X = \frac{1}{4}g^2(u^2+\omega^2).$$

The four components $A_{3\mu}, A_{8\mu}, B_\mu, C_\mu$ are mixing. The squared-mass matrix is obtained as

$$\frac{g^2}{2}\begin{pmatrix} \frac{1}{2}(u^2+v^2) & \frac{u^2-v^2}{2\sqrt{3}} & -\frac{t_1(u^2+2v^2)}{3} & \frac{t_2(u^2-v^2)}{3} \\ \frac{u^2-v^2}{2\sqrt{3}} & \frac{1}{6}(u^2+v^2+4\omega^2) & -\frac{t_1(u^2-2(v^2+\omega^2))}{3\sqrt{3}} & \frac{t_2(u^2+v^2+4\omega^2)}{3\sqrt{3}} \\ -\frac{t_1(u^2+2v^2)}{3} & -\frac{t_1(u^2-2(v^2+\omega^2))}{3\sqrt{3}} & \frac{2}{9}t_1^2(u^2+4v^2+\omega^2) & -\frac{2}{9}t_1t_2(u^2-2(v^2+\omega^2)) \\ \frac{t_2(u^2-v^2)}{3} & \frac{t_2(u^2+v^2+4\omega^2)}{3\sqrt{3}} & -\frac{2}{9}t_1t_2(u^2-2(v^2+\omega^2)) & \frac{2}{9}t_2^2(u^2+v^2+4(\omega^2+9\Lambda^2)) \end{pmatrix},$$

where $t_1 \equiv g_X/g$, $t_2 \equiv g_N/g$.

The four components $A_{3\mu}, A_{8\mu}, B_\mu, C_\mu$ are mixing. The squared-mass matrix is obtained as

$$\frac{g^2}{2}\begin{pmatrix} \frac{1}{2}(u^2+v^2) & \frac{u^2-v^2}{2\sqrt{3}} & -\frac{t_1(u^2+2v^2)}{3} & \frac{t_2(u^2-v^2)}{3} \\ \frac{u^2-v^2}{2\sqrt{3}} & \frac{1}{6}(u^2+v^2+4\omega^2) & -\frac{t_1(u^2-2(v^2+\omega^2))}{3\sqrt{3}} & \frac{t_2(u^2+v^2+4\omega^2)}{3\sqrt{3}} \\ -\frac{t_1(u^2+2v^2)}{3} & -\frac{t_1(u^2-2(v^2+\omega^2))}{3\sqrt{3}} & \frac{2}{9}t_1^2(u^2+4v^2+\omega^2) & -\frac{2}{9}t_1t_2(u^2-2(v^2+\omega^2)) \\ \frac{t_2(u^2-v^2)}{3} & \frac{t_2(u^2+v^2+4\omega^2)}{3\sqrt{3}} & -\frac{2}{9}t_1t_2(u^2-2(v^2+\omega^2)) & \frac{2}{9}t_2^2(u^2+v^2+4(\omega^2+9\Lambda^2)) \end{pmatrix},$$

where $t_1 \equiv g_X/g$, $t_2 \equiv g_N/g$.

The mass mixing matrix of $A_{3\mu}, A_{8\mu}, B_\mu, C_\mu$ contains one exact eigenvalues with the corresponding eigenstates as follows

$$M_\gamma^2 = 0, A_\mu = \frac{\sqrt{3}}{\sqrt{3+4t_1^2}}\left(t_1 A_{3\mu} - \frac{t_1}{\sqrt{3}}A_{8\mu} + B_\mu\right).$$

A_μ is identified as photon. The new gauge boson C_μ does not give any contribution to the photon field.

The vertex coefficient of $\bar{e}\gamma^{\mu}eA_{\mu}$ is identified as the electromagnetic gauge coupling constant, we get

$$t_1 = \frac{\sqrt{3}s_W}{\sqrt{3 - 4s_W^2}}.$$

$(A_{3\mu}, A_{8\mu}, B_\mu, C_\mu) \to (A_\mu, Z_\mu, Z'_\mu, C_\mu)$, where

$$Z_\mu = \frac{\sqrt{3+t_1^2}}{\sqrt{3+4t_1^2}} A_{3\mu} + \frac{t_1(\sqrt{3}t_1 A_{8\mu} - 3B_\mu)}{\sqrt{3+t_1^2}\sqrt{3+4t_1^2}}, \quad Z'_\mu = \frac{\sqrt{3}}{\sqrt{3+t_1^2}} A_{8\mu} + \frac{t_1}{\sqrt{3+t_1^2}} B_\mu.$$

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$$Z_\mu = \frac{\sqrt{3+t_1^2}}{\sqrt{3+4t_1^2}} A_{3\mu} + \frac{t_1(\sqrt{3}t_1 A_{8\mu} - 3B_\mu)}{\sqrt{3+t_1^2}\sqrt{3+4t_1^2}}, \quad Z'_\mu = \frac{\sqrt{3}}{\sqrt{3+t_1^2}} A_{8\mu} + \frac{t_1}{\sqrt{3+t_1^2}} B_\mu.$$

Z_μ, Z'_μ, C_μ are mixing

$$\frac{g^2}{2}\begin{pmatrix} \frac{(3+4t_1^2)(u^2+v^2)}{2(3+t_1^2)} & -\frac{\sqrt{3+4t_1^2}((-3+2t_1^2)u^2+(3+4t_1^2)v^2)}{6(3+t_1^2)} & \frac{\sqrt{3+4t_1^2}t_2(u^2-v^2)}{3\sqrt{3+t_1^2}} \\ -\frac{((-3+2t_1^2)u^2+(3+4t_1^2)v^2)}{6(3+t_1^2)[\sqrt{3+4t_1^2}]^{-1}} & \frac{(3-2t_1^2)^2u^2+(3+4t_1^2)^2v^2+4(3+t_1^2)^2\omega^2}{18(3+t_1^2)} & \frac{t_2((3-2t_1^2)u^2+(3+4t_1^2)v^2+4(3+t_1^2)\omega^2)}{9\sqrt{3+t_1^2}} \\ \frac{\sqrt{3+4t_1^2}t_2(u^2-v^2)}{3\sqrt{3+t_1^2}} & \frac{t_2((3-2t_1^2)u^2+(3+4t_1^2)v^2+4(3+t_1^2)\omega^2)}{9\sqrt{3+t_1^2}} & \frac{2}{9}t_2^2(u^2+v^2+4(\omega^2+9\Lambda^2)) \end{pmatrix}.$$

In the limit $\Lambda \sim \omega$

$$\begin{aligned}
Z_{1\mu} &= Z_\mu, \quad m^2_{Z^1} \simeq \frac{g^2(u^2+v^2)}{4c^2_W}, \\
Z_{2\mu} &= \cos\xi \mathcal{Z}'_\mu + \sin\xi \mathcal{C}_\mu, \quad Z_{N\mu} = -\sin\xi \mathcal{Z}'_\mu + \cos\xi \mathcal{C}_\mu, \\
m^2_{Z_N} &\simeq \frac{g^2}{18}\Big((3+t_1^2)\omega^2 + 4t_2^2(\omega^2+9\Lambda^2) \\
&+ \sqrt{((3+t_1^2)\omega^2 - 4t_2^2(\omega^2+9\Lambda^2))^2 + 16(3+t_1^2)t_2^2\omega^4}\Big) + \mathcal{O}(\frac{u^2, v^2}{\omega^2, \Lambda^2}), \\
m^2_{Z_2} &\simeq \frac{g^2}{18}\Big((3+t_1^2)\omega^2 + 4t_2^2(\omega^2+9\Lambda^2) \\
&- \sqrt{((3+t_1^2)\omega^2 - 4t_2^2(\omega^2+9\Lambda^2))^2 + 16(3+t_1^2)t_2^2\omega^4}\Big) + \mathcal{O}(\frac{u^2, v^2}{\omega^2, \Lambda^2}),
\end{aligned}$$

where $\tan 2\xi = \frac{4\sqrt{3+t_1^2}t_2\omega^2}{(3+t_1^2)\omega^2 - 4t_2^2(\omega^2+9\Lambda^2)}$.

In the limit $\Lambda \gg \omega$

$$
\begin{aligned}
Z_{N\mu} &\sim C_\mu, \quad m^2_{Z_N} \simeq 4g^2 t_2^2 \Lambda^2, \\
Z_{1\mu} &= \cos\xi Z_\mu - \sin\xi Z'_\mu, \quad Z_{2\mu} = \sin\xi Z_\mu + \cos\xi Z'_\mu, \\
m^2_{Z_1} &\simeq \frac{g^2}{8}\left(u^2 + \omega^2 + \frac{u^2 + 4v^2 + \omega^2}{3 - 4s_W^2}\right. \\
&- \left. 4\frac{\sqrt{c_W^4 u^4 + v^4 - c_{2W} v^2 \omega^2 + c_W^4 \omega^4 + u^2(-c_{2W} v^2 + (-1 + 2s_W^4)\omega^2)}}{(3 - 4s_W^2)}\right), \\
m^2_{Z_2} &\simeq \frac{g^2}{8}\left(u^2 + \omega^2 + \frac{u^2 + 4v^2 + \omega^2}{3 - 4s_W^2}\right. \\
&+ \left. 4\frac{\sqrt{c_W^4 u^4 + v^4 - c_{2W} v^2 \omega^2 + c_W^4 \omega^4 + u^2(-c_{2W} v^2 + (-1 + 2s_W^4)\omega^2)}}{(3 - 4s_W^2)}\right),
\end{aligned}
$$

where

$$
\tan 2\xi = \frac{\sqrt{3 - 4s_W^2}(c_{2W} u^2 - v^2)}{((-1 + 2s_W^4)u^2 - c_{2W} v^2 + 2c_W^4 \omega^2)},
$$

If we assume $\omega \gg u, v$, then $\tan 2\xi \to 0$. We get

$$Z^1_\mu \sim Z_\mu, \quad m^2_{Z^1} \simeq \frac{g^2(u^2+v^2)}{4c^2_W},$$
$$Z^2_\mu \sim Z'_\mu, \quad m^2_{Z^2} \simeq \frac{c^2_{2W}u^2+v^2+4c^4_W\omega^2}{(3-4s^2_W)c^2_W}.$$

The gauge boson Z^1_μ is identified as Z_μ in the SM.

Fermions

The Dirac masses are written in the form $-\bar{f}_L m_f f_R + \text{H.c.}$ From the $\mathcal{L}_{\text{Yukawa}}$, we obtain

$$\begin{aligned} m_U &= -\frac{1}{\sqrt{2}} h^U \omega, \quad [m_D]_{\alpha\beta} = -\frac{1}{\sqrt{2}} h^D_{\alpha\beta} \omega, \\ [m_u]_{\alpha a} &= \frac{1}{\sqrt{2}} h^u_{\alpha a} v, \quad [m_u]_{3a} = -\frac{1}{\sqrt{2}} h^u_a u, \\ [m_d]_{\alpha a} &= -\frac{1}{\sqrt{2}} h^d_{\alpha a} v, \quad [m_d]_{3a} = -\frac{1}{\sqrt{2}} h^d_a v, \\ [m_e]_{ab} &= -\frac{1}{\sqrt{2}} h^e_{ab} v, \quad [m^D_\nu]_{ab} = -\frac{1}{\sqrt{2}} h^\nu_{ab} u. \end{aligned}$$

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m_U &= -\frac{1}{\sqrt{2}} h^U \omega, \quad [m_D]_{\alpha\beta} = -\frac{1}{\sqrt{2}} h^D_{\alpha\beta}\omega, \\
[m_u]_{\alpha a} &= \frac{1}{\sqrt{2}} h^u_{\alpha a} v, \quad [m_u]_{3a} = -\frac{1}{\sqrt{2}} h^u_a u, \\
[m_d]_{\alpha a} &= -\frac{1}{\sqrt{2}} h^d_{\alpha a} v, \quad [m_d]_{3a} = -\frac{1}{\sqrt{2}} h^d_a v, \\
[m_e]_{ab} &= -\frac{1}{\sqrt{2}} h^e_{ab} v, \quad [m^D_\nu]_{ab} = -\frac{1}{\sqrt{2}} h^\nu_{ab} u.
\end{aligned}$$

The right handed neutrinos get Majorana masses in the form $-\frac{1}{2}\bar{\nu}^c_R m^M_\nu \nu_R + \text{H.c.}$, where

$$[m^M_\nu]_{ab} = -\sqrt{2} h'^\nu_{ab} \Lambda.$$

The observed neutrinos ($\sim \nu_L$) naturally get small masses via a type I seesaw mechanism,

$$m^{\text{eff}}_\nu = -m^D_\nu (m^M_\nu)^{-1} (m^D_\nu)^T \sim \frac{(h^\nu)^2}{h'^\nu}\frac{u^2}{\Lambda}.$$

The masses of N_R can be generated via an effective operator invariant under the 3-3-1-1 symmetry

$$\frac{\lambda_{ab}}{M}\bar{\psi}^c_{aL}\psi_{bL}(\chi\chi)^* + \text{H.c},$$
$$[m_{N_R}]_{ab} = -\lambda_{ab}\frac{\omega^2}{M}.$$

Assume that $M \sim \omega$ then $m_{N_R} \sim \omega$.

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▷ After spontaneous symmetry breaking, there are

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- 9 massive gauge bosons $Z^N, Z^1, Z^2, X^0, X^{0*}, Y^{\pm}, W^{\pm}$, and one massless γ,
- 4 neutral Higgs bosons H, H_1, H_2, H_3, one massive pseudoscalar A, complex Higgs H', H'^*, 4 charged scalars $H^{\pm}_4, H^{\pm}_5$.

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- ▷ In the limit $\Lambda \gg \omega$, H_3, Z^N, ν_R have mass in order $\mathcal{O}(\Lambda)$.
  All other new particles, $A, H_1, H_2, H^\pm_4, H^\pm_5, H', H'^*$, $Z^2_\mu, X^0_\mu, X^{0*}_\mu, Y^\pm_\mu$, U, D_α, N_R, have mass in order $\mathcal{O}(\omega)$.

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▷ In this model, $L(G_X, H'^*, H^-_4, G^-_Y, X^0, Y^-) = 1$ while the remaining Higgs and gauge bosons have zero lepton number.

▷ $P(N_R, X, Y, U, D, H_4, H') = -1$ and all other particles have $P = +1$. The lightest and neutral particle among odd parity particles can be a dark matter candidate.

Thanks you!