QCD axions with high scale inflation

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Outline

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* Cosmological constraints on the QCD axion
  
  Before BICEP2 & After BICEP2
  (Assume the detection of primordial gravitational waves)

* Implications (for string theoretic realization of the QCD axion)

* Conclusion
Strong CP problem

Fine tuning problem for the CP violating sector of the SM:

\[ \frac{\theta_{\text{QCD}}}{32\pi^2} G_{a\mu\nu} \tilde{G}_a^{\mu\nu} + (y_q H q_L q_R + h.c) \]

\[ \Rightarrow \bar{\theta} = \theta_{\text{QCD}} + \text{Arg} \cdot \text{Det}(y_q) \quad \text{(CP violation in strong interactions)} \]

\[ \delta_{\text{KM}} \sim \text{Arg}(y_q) \quad \text{(CP violation in weak interactions)} \]

Neutron EDM:

\[ d_n \sim 10^{-16} \bar{\theta} \text{ e \cdot cm} < 10^{-26} \text{ e \cdot cm} \]

\[ \Rightarrow |\bar{\theta}| = |\theta_{\text{QCD}} + \text{Arg} \cdot \text{Det}(y_q)| < 10^{-10} \]

CP violation in the weak interactions

\[ \delta_{\text{KM}} \sim 1 \]

Why \( |\theta_{\text{QCD}} + \text{Arg Det}(y_q)| < 10^{-10} \), while \( \delta_{\text{KM}} \sim \text{Arg}(y_q) \sim 1 \)?
Strong CP problem

Why $|\theta_{\text{QCD}} + \text{Arg } \det(y_q)| < 10^{-10}$, while $\delta_{\text{KM}} \sim \text{Arg}(y_q) \sim 1$?

Unlike the gauge hierarchy problem, anthropic argument cannot explain this puzzle.

It is thus likely that there should be some physical explanation for the absence of CP violation in the strong interactions.
Axion solution

Introduce a spontaneously broken anomalous global U(1) symmetry  
(Peccei-Quinn symmetry)

⇒ \( \theta_{\text{QCD}} \) becomes a dynamical field "axion"

= Nambu-Goldstone boson of the spontaneously broken U(1)_PQ

\[
\frac{1}{32\pi^2} (\theta_{\text{QCD}} + \text{Arg} \cdot \text{Det}(y_q)) \, G^{a\mu\nu} \tilde{G}^a_{\mu\nu} = \frac{1}{32\pi^2} \frac{\langle a \rangle}{f_a} \, G^{a\mu\nu} \tilde{G}^a_{\mu\nu}
\]

\( f_a \) = Axion scale = Mass scale of the spontaneous breaking of U(1)_PQ  
(Axion decay constant)

Low energy QCD dynamics develops an axion potential minimized at \( \langle a \rangle = 0 \):

QCD becomes CP conserving once the axion is settled down at its VEV.
Key assumption:

The origin of $V_{QCD}(a)$ is the explicit PQ breaking by the QCD anomaly.

Generically PQ symmetry can be explicitly broken also by UV physics such as quantum gravity effects:

$$\partial_{\mu}J_{\mu PQ} = \frac{g_c^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a + \Delta_{UV}$$

There is no reason that these two axion potentials have a common minimum.

To achieve $|\langle a \rangle/f_a| = |\theta_{QCD} + \text{Arg} \cdot \text{Det}(y_q)| < 10^{-10}$, the explicit PQ breaking other than the QCD anomaly should be highly suppressed as

$$\Delta V(a) < 10^{-10} f_\pi^2 m_\pi^2 \sim 10^{-14} \text{GeV}^4$$
Q1: What is the origin of such PQ symmetry which is extremely well protected from global-symmetry-breaking UV physics? 

(Any connection to gauge symmetries in the UV limit?)

To complete the solution, this question needs to be addressed within the framework of fundamental theory, e.g. string theory.

Q2: What is the physical mechanism to determine \( f_a \)? 

(Any connection to the other symmetry breakings such as compactification, SUSY breaking, ...?)
Most of axion physics is determined by the axion scale $f_a$:

* **QCD axion mass:** $m_a \sim 5 \times 10^{-6} \left( \frac{10^{12} \text{GeV}}{f_a} \right) \text{eV}$

* **QCD axion-photon couplings:** $g_{a\gamma\gamma} \sim 10^{-15} \left( \frac{10^{12} \text{GeV}}{f_a} \right) \text{GeV}^{-1}$

Applying this form of $g_{a\gamma\gamma}$ for the star cooling by axion emission

$\Rightarrow \quad f_a > 4 \times 10^8 \text{ GeV}$

$\Rightarrow \quad \tau_a \gg 10^{17} \text{ sec},$ so once axions were produced in the early universe, they constitute (part of) the DM in the present universe.
Relic abundance of the QCD axion dark matter:

Misalignment + Axionic topological defect (strings, walls)

Preskill, Wise, Wilczek ‘83; Abbott, Sikivie ‘83; Dine, Fischler ‘83; ...
+ Davis ‘86; Davis, Harari, Sikivie ‘87; Davis, Shellard ‘89; Lyth ‘92; ...

Initial axion field misaligned from the minimum

DM axions from both misalignment and topological defects are produced when $m_a(t) \propto 1/f_a \sim H(t)$, and then evolve like non-relativistic matter

$$\Omega_a \sim 0.2 \left( \theta_0^2 + \delta \theta^2 + R_{\text{defect}} \right) \left( \frac{f_a(t_0)}{10^{12} \text{ GeV}} \right)^{7/6} \quad (f_a < \mathcal{O}(10^{17}) \text{ GeV})$$

For axions produced by topological defects $\sim 40 – 120$
QCD axion has a good potential to be experimentally tested!

Axion dark matter

Astrophysical bound

$f_a$ $10^9$ $10^{10}$ $10^{11}$ $10^{12}$ $10^{13}$ $10^{15}$ $10^{16}$ $10^{17}$ $10^{18}$ (GeV)

Axion mediated force (Arvanitaki & Geraci '14)

Resonant conversion of DM axions to microwave photons (Sikivie '83)

Resonant spin flip with oscillating EDMs induced by oscillating DM axions (Graham & Rajendran '13)
Ongoing experiment which has a potential to detect axion DM:  
**Axion Dark Matter eXperiment (ADMX)**

A new research center for axion DM search has been launched recently in Korea:  **The IBS Center for Axion and Precision Physics (CAPP)**

One of the major scientific goals of **CAPP** (in 6-8 years):

**ADMX-type cavity experiment searching for axion DM with**

$$m_a \sim 10 - 80 \mu eV \ (f_a \sim 6 \times 10^{10} - 5 \times 10^{11} \text{ GeV})$$
**ADMX vs CAPP** (Y. Semertzidis)

Early stage

B-field 25 T

High-Q $10^7$

B-field 35 T
Cosmological constraints on the QCD axions

In principle, there can be many different cosmological scenarios which result in different forms of constraints.

Generically $f_a$ and $m_a$ can depend on some field variables (e.g. moduli, saxion, QCD dilaton, ...), and therefore can have nontrivial cosmological evolution from the inflation epoch to the present time:

* $f_a(t_I) \sim f_a(t_0)$ or $f_a(t_I) \gg f_a(t_0)$ or $f_a(t_I) = 0$

* $m_a(t_I) \ll H(t_I)$ or $m_a(t_I) \geq H(t_I)$

There can be also a late entropy production, which would affect the axion cosmology.
Which cosmological scenario is more plausible than the others depends on

Q1: What is the UV origin of the PQ symmetry?
   PQ symmetry is required to be well protected from UV physics such as quantum gravity.

Q2: What is the physical mechanism to determine \( f_a \)?
   Compactification? SUSY breaking? New dynamical mass scale? ...

Here we will focus on relatively simple scenarios in which

* Explicit PQ breaking other than the QCD anomaly is suppressed enough over the period from inflation to the QCD phase transition:

\[ m_a(t) \ll H(t) \] before the axion DM are produced around the QCD phase transition.

(Explicit PQ breaking other than the QCD anomaly at present \( < 10^{-10} \times \) QCD anomaly.)

* There is no late entropy production after the QCD phase transition.

* \( f_a(t_I) \) can be generically different from \( f_a(t_0) \)
Scenario A:

PQ symmetry is non-linearly realized (spontaneously broken) during inflation, and never restored thereafter.

No axionic strings or domain walls, but the axion field could have a nonzero misalignment together with a fluctuation generated during the inflation period:

\[ \delta \theta = \frac{\delta a(t)}{f_a(t)} = \frac{H(t)}{2\pi f_a(t)} \]

\[ \langle a \rangle = 0 \]

\[ \theta_0 \] = Free parameter in the range \([−\pi, \pi]\)

\[ \delta a(t) = \frac{H(t)}{2\pi} \] = axion fluctuation generated during inflation

\[ f_a(t) \] = Axion scale during inflation period,

which can be generically different from \( f_a(t_0) \)
Scenario A:

Relic axion dark matter:

\[
\Omega_a \sim 0.2 \left( \theta_0^2 + \delta \theta^2 \right) \left( \frac{f_a(t_0)}{10^{12} \text{ GeV}} \right)^{7/6} \leq \Omega_{DM} \simeq 0.24 \\
( f_a < \mathcal{O}(10^{17}) \text{ GeV} )
\]

Axion isocurvature perturbation: 

\[
\frac{\deltalerm{n_a}/s}{\delta \theta} \neq 0 \quad \left( \delta \theta \sim \frac{H(t_I)}{2\pi f_a(t_I)} \right)
\]

Axenides, Brandenberger, Turner ‘83; Seckel, Turner ‘85; Linde ‘85; Fox, Pierce, Thomas ‘04; ...

\[
\frac{\delta T}{T}_{iso} \sim \frac{\delta \rho_a}{\rho_{DM}} \sim \frac{\Omega_a}{\Omega_{DM}} \frac{\delta \rho_a}{\rho_a} \sim \frac{\Omega_a}{\Omega_{DM}} \frac{2 \delta \theta}{\theta_0} \quad \left( \rho_a \propto \langle \theta_{mis}^2 \rangle, \ \delta \theta \ll \theta_0 \right)
\]

\[
\sim \left( \frac{\Omega_a}{\Omega_{DM}} \right)^{1/2} \left( \frac{f_a(t_0)}{10^{12} \text{ GeV}} \right)^{7/12} \left( \frac{H(t_I)}{\pi f_a(t_I)} \right) < 0.2 \left( \frac{\delta T}{T} \right)_{tot} \sim 2 \times 10^{-6}
\]

(PLANCK)
Scenario B: the last spontaneous PQ breaking occurred after inflation

There are PQ strings attached by $N_{DW}$ domain walls, which cause cosmological domain wall problem unless $N_{DW} = 1$:

Axionic string produced during the PQ phase transition

Axionic string attached by walls produced during the QCD phase transition

$$\frac{a}{f_a} \rightarrow \frac{a}{f_a} + 2\pi$$

$$N_{DW} = 2$$

$$N_{DW} = \sum_i q_i \text{Tr}(T_c^2(\psi_i)) = \text{nonzero integer}$$

$$\cos\left(\frac{a}{f_a}\right) \rightarrow \cos\left(\frac{N_{DWA}}{f_a}\right)$$

⇒ Axion domain-wall number = $N_{DW} = \sum_i q_i \text{Tr}(T_c^2(\psi_i)) = 1$
Scenario B:

No isocurvature perturbation, but axion dark matters can be produced by collapsing string-wall system with \( N_{DW} = 1 \), as well as by the coherent oscillation of misaligned axion field:

\[
\Omega_a \sim 0.2 \left( \langle \theta_{mis}^2 \rangle + R_{\text{defect}} \right) \left( \frac{f_a(t_0)}{10^{12} \text{ GeV}} \right)^{7/6}
\]

* Present horizon involves many different patches which were casually disconnected at the moment of the PQ phase transition:

\[
\langle \theta_{mis}^2 \rangle = \frac{\pi^2}{3}
\]

* Numerical simulation: \( R_{\text{defect}} \sim 40 - 120 \) \quad \text{Hiramatsu et al, '12}

\[
\Rightarrow \quad \Omega_a \sim (10-25) \times \left( \frac{f_a(t_0)}{10^{12} \text{ GeV}} \right)^{7/6} \leq 0.24
\]

\[
\Rightarrow \quad 4 \times 10^8 \text{ GeV} < f_a < (2-4) \times 10^{10} \text{ GeV}
\]
A frequently used summary of the constraints:

\[ f_a(t_1) = f_a(t_0) \ \& \ \Omega_a = \Omega_{\text{DM}} \text{ for Scenario A} \]

Scenario A

Unshaded regions are allowed.

Scenario B

Hertzberg, Tegmark, Wilczek ’08; Visinelli, Gondolo ’09, 14; Wanz, Shellard ’10; Marsh et al ’14...
Summary for more generic situation: \( f_a(t_I) \neq f_a(t_0), \, \Omega_a/\Omega_{DM} \leq 1 \) (before BICEP2)

\[
\Omega_a/\Omega_{DM} \sim (\delta \theta^2 + \delta \theta^2) \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{7/6}
\]

Anthropic axion DM in Scenario A
Linde '88
Tegmark et al '06
Freivogel '08

Isocurvature constraint for \( \delta \theta = 10^{-7} \)

\[
\delta \theta \equiv \frac{\delta a(t_I)}{f_a(t_I)} = \frac{H(t_I)}{2\pi f_a(t_I)}
\]
After BICEP2: \( H(t_1) \simeq 10^{14} \text{ GeV} \)

\[
\frac{\delta \theta}{2 \pi f_a(t_1)} = \frac{10^{14} \text{ GeV}}{2 \pi f_a(t_1)}
\]

KC, Jeong, Seo,
arXiv:1404.3880

The PQ symmetry should be either restored (Scenario B), or spontaneously broken at much higher scale during inflation (Scenario A):

\[
f_a(t_1) = 0 \quad \text{or} \quad f_a(t_1) \gg f_a(t_0)
\]
What would be the most probable parameter region for the QCD axion compatible with high scale inflation?

* For Scenario B *(PQ symmetry restored during inflation)*, if one tries to get such a PQ symmetry from top-down approach, e.g. within the framework of string theory, usually one finds $N_{DW} > 1$. ⇒ Take Scenario A.

* Perturbative axion coupling can not be significantly weaker than the gravitational interaction *(Weak gravity conjecture):* Arkani-Hamed, Motl, Nicolis, Vafa ’07

$$\frac{g^2}{32\pi^2f_a} aG \tilde{G} \quad \Rightarrow \quad f_a(t_1) \leq \mathcal{O}\left(\frac{g^2}{8\pi^2 M_{Pl}}\right) \quad \Rightarrow \quad f_a(t_1) \leq 10^{17} \text{ GeV}$$

* Accept the tuning of $\theta_0$ if there is an anthropic reasoning for the tuning, but no more tuning than that.
Most probable region:

\[ \theta_0 \sim 0.1 - \mathcal{O}(1) \]
\[ f_a(t_0) \sim 10^9 - 10^{11} \text{ GeV} \]
\[ f_a(t_I) \sim 10^{15} - 10^{17} \text{ GeV} \]

\[ \frac{\Omega_a}{\Omega_{DM}} \leq \mathcal{O}(0.1) \]
Regardless of whether BICEP2 detected a signal of primordial gravitational waves or just dusts, similar conclusion applies for generic high scale inflation scenario (chaotic inflation) which predicts

\[ r = 0.01 - 0.2 \quad (H(t_f) \sim 10^{13} - 10^{14} \text{ GeV}) \]
Implications:

The big difference between $f_a(t_0) \sim 10^9 - 10^{11}$ GeV and $f_a(t_I) \sim 10^{15} - 10^{17}$ GeV suggests that the axion scale is generated by SUSY breaking effects:

KC, Jeong, Seo, arXiv:1404.3880

\[ V_{PQ} = -m^2_\phi |\phi|^2 + \lambda |\phi|^4 + \ldots \quad \text{or} \quad -m^2_\phi |\phi|^2 + \frac{\lambda |\phi|^6}{M^2_{Pl}} + \ldots \]

\[ m^2_\phi(t_0) \sim m^2_{3/2}(t_0) \]
\[ m^2_\phi(t_I) \sim H^2(t_I) \]

\[ f_a(t_0) \sim \langle \phi(t_0) \rangle \sim m_{3/2}(t_0) \text{ or } \sqrt{m_{3/2}(t_0)M_{Pl}} \sim 10^9 - 10^{11} \text{ GeV} \]
\[ f_a(t_I) \sim \langle \phi(t_I) \rangle \sim H(t_I) \text{ or } \sqrt{H(t_I)M_{Pl}} \sim 10^{14} - 10^{16} \text{ GeV} \]

\( m_{\text{higgs}} \sim 125 \text{ GeV} \quad \Rightarrow \quad m_{\text{SUSY}} \sim m_{3/2} \sim 10^3 - 10^9 \text{ GeV} \)
Realization of such QCD axion in string theory

4D effective theory of string compactification generically involves axion-like fields $a_{st}$ originating from higher-dim antisymmetric tensor gauge fields. \( \text{Witten '84} \)

* Origin of PQ symmetry well protected from quantum gravity:

**Gauge-axion unification** (with extra dimension)

$\Rightarrow$ 4-dim perturbative shift symmetry: $U(1)_{\text{shift}} : a_{st} \rightarrow a_{st} + \text{constant}$

($a_{st} =$ zero mode of higher-dim antisymmetric tensor gauge field)

which is a low energy remnant of higher-dim gauge symmetry

* Axion scales: $\frac{8\pi^2 f_a}{g^2} \sim \text{String or the Planck scale}$

$\Rightarrow f_a(t_0) \sim f_a(t_I) \sim 10^{15} - 10^{17} \text{ GeV}$ \( \text{KC, Kim '85; Svrcek, Witten '06} \)

However this simple realization of the QCD axion in string theory is in trouble with the isocurvature constraint in high scale inflation scenario.
String theory admits a simple generalization involving anomalous U(1)A gauge symmetry, in which an intermediate axion scale is generated by SUSY breaking effect as desired, while the compactification scale is close to the Planck scale:

* Shift symmetry from higher-dim gauge symmetry:

\[ U(1)_{\text{shift}} : a_{st} \rightarrow a_{st} + \text{constant} \]

* Anomalous U(1) gauge symmetry under which \( a_{st} \) implements the Green-Schwarz anomaly cancellation mechanism:

\[ U(1)_A : A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x), \quad a_{st} \rightarrow a_{st} + \delta_{GS} \alpha(x), \quad \phi_i \rightarrow e^{iq_i \alpha(x)} \phi_i \]

\[ \left( \delta_{GS} = \frac{1}{8\pi^2} \sum_i q_i \text{Tr}(T_a^2(\phi_i)) = \mathcal{O} \left( \frac{1}{8\pi^2} \right) \right) \]

\[ \Rightarrow \quad \text{Generically } U(1)_{PQ} = \text{Combination of } U(1)_{\text{shift}} \text{ and } U(1)_A \]

QCD axion = Combination of \( a_{st} \) and \( \arg(\phi) \)
In many cases, such model admits a vacuum configuration

\[ \text{Moduli-dependent Fayet-Illiopoulos (FI) term} = U(1)_A \text{ charged matter fields} = 0 \text{ with unbroken N}=1 \text{ SUSY} \]

(Such vacuum solution is quite common in Type II string models with D-branes and also heterotic string models with background U(1) gauge bundles.)

In this limit, \( a_{st} \) is eaten by the \( U(1)_A \) gauge boson, while leaving

an unbroken global \( PQ \) symmetry = global part of \( U(1)_A \) without \( a_{st} \) as a low energy remnant.

Such supersymmetric solution with a linearly realized \( PQ \) symmetry can be destabilized by \( U(1)_A \) D-term induced tachyonic SUSY breaking mass of \( U(1)_A \)-charged matter field, yielding

\[ f_a \sim \langle \phi \rangle \sim m_\phi \text{ or } \sqrt{m_\phi M_{Pl}} \quad (m_\phi^2 \sim q_\phi D_A < 0) \]
An effective 4D SUGRA analysis: KC, Jeong, Seo, arXiv:1404.3880

U(1)$_A$ & PQ breaking sector:

\[ K = \frac{c_0^2 M_{Pl}^2}{2} (\tau - \tau_0 - \delta_{GS} V_A)^2 + \phi_1^* e^{-V_A} \phi_1 + \phi_2^* e^{(n+2)V_A} \phi_2, \]
\[ W = \lambda \frac{\phi_1^n \phi_2^2}{M_{Pl}^n}, \quad \left| \frac{\partial K_0}{\partial \tau} \right|_{\tau = \tau_0} = 0 \quad c_0^2 = \left| \frac{\partial^2 K_0}{\partial \tau^2} \right|_{\tau = \tau_0} \]

$(\tau, a_{st}) = \text{(modulus, axion)}$ for the Green-Schwarz mechanism

$\phi_i = U(1)_A$-charged matter fields

SUSY breaking + Inflaton sector: Kawasaki, Yamaguchi, Yanagida '00, Linde, Kallosh '10, ...

(Marchesano, Uranga, Shiu '14: F-term axion monodromy)

Couplings between the two sectors:

\[ \Delta K = (k |Z|^2 + \kappa |X|^2)(\tau - \tau_0 - \delta_{GS} V_A) + \frac{k_i |Z|^2 + \kappa_i |X|^2}{M_{Pl}^2} \phi_i^* e^{-q_i V_A} \phi_i \]
Tachyonic soft mass of $\phi_1$ from the $U(1)_A$ D-term at both the present time and the inflationary epoch:

$$m_{\phi_1}^2 = q_1 D_A + \mathcal{O} \left( \frac{|F^Z|^2}{M_{Pl}^2}, \frac{|F^X|^2}{M_{Pl}^2} \right)$$

$$D_A \sim \frac{1}{\delta_{GS}} \left( k \frac{|F^Z|^2}{M_{Pl}^2} + \kappa \frac{|F^X|^2}{M_{Pl}^2} \right) \sim \frac{16\pi^2}{g^2} \left( m_{3/2}^2(t_0) + H^2(t_1) \right)$$

$$\delta_{GS} \sim \mathcal{O} \left( \frac{g^2}{8\pi^2} \right)$$

$k, \kappa =$ Kähler potential couplings between the $U(1)_A$ sector and the SUSY breaking sector $\sim \mathcal{O}(1)$

$$\frac{F^Z}{M_{Pl}} \sim m_{3/2}(t_0), \quad \frac{F^X}{M_{Pl}} \sim H(t_1)$$

$$f_a(t_0) \sim \langle \phi_1(t_0) \rangle \sim 4\pi m_{3/2} \text{ or } \sqrt{4\pi m_{3/2} M_{Pl}} \sim 10^9 - 10^{11} \text{ GeV}$$

$$f_a(t_1) \sim \langle \phi_1(t_1) \rangle \sim 4\pi H(t_1) \text{ or } \sqrt{4\pi H(t_1) M_{Pl}} \sim 10^{15} - 10^{17} \text{ GeV}$$
Conclusion

* In high scale inflation scenario, the QCD axion scale and the relic axion abundance are severely constrained by the isocurvature perturbation bound.

The most probable scenario for the QCD axion compatible with high scale inflation is likely to be

\[
\begin{align*}
f_a(t_0) &\sim 10^9 - 10^{11} \text{ GeV} \\
f_a(t_I) &\sim 10^{15} - 10^{17} \text{ GeV} \\
\frac{\Omega_a}{\Omega_{DM}} &\leq O(0.1)
\end{align*}
\]

This suggests that the spontaneous breakdown of PQ symmetry is triggered by SUSY-breaking effects, leading to a specific connection between the axion scale and the SUSY breaking scale as

\[
\begin{align*}
f_a(t_0) &\sim m_{3/2} \text{ or } \sqrt{m_{3/2}\mpl} \sim 10^9 - 10^{11} \text{ GeV} \\
f_a(t_I) &\sim H(t_I) \text{ or } \sqrt{H(t_I)\mpl} \sim 10^{15} - 10^{17} \text{ GeV}
\end{align*}
\]
* Compactified string models involving an anomalous U(1) gauge symmetry with vanishing FI-term provide an appealing setup to realize such scenario.

Those models explain the origin of PQ symmetry which is protected well from quantum gravity effects, while giving an intermediate axion scale generated by D-term SUSY breaking in both the present Universe and the inflationary epoch.

* This scenario can be tested by axion-mediated force and more sensitive resonant cavity experiment for axion DM ($\Omega_a/\Omega_{DM} \sim 0.01 - 0.1$).

\[ f_a \quad 10^9 \quad 10^{10} \quad 10^{11} \quad 10^{12} \quad 10^{13} \quad \text{(GeV)} \]