# Searching for general relativistic signatures on large scales

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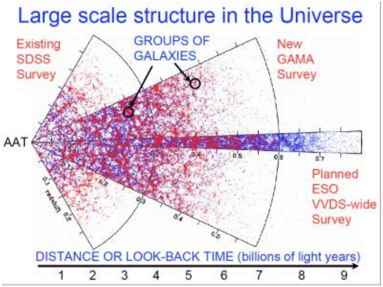
Based on collaborations with S. G. Biern, J.-c. Hwang, D. Jeong and H. Noh

# Outline

# Why GR in LSS?

Planned galaxy surveys: DESI, HETDEX, LSST, Euclid, WFIRST...




Larger and larger volumes, eventually accessing the scales comparable to the horizon: beyond Newtonian gravity, fully general relativistic approach (or any modification) is necessary

# Why non-linearity and gauge in LSS?

- Non-linearity is prominent in large scale structure thus accurate modeling of of non-linearity is very important
- GR is a gauge theory, thus observational quantities only make sense after choosing the coordinate systems

On large scales where non-linearity can be probed by observations with improved accuracy, density contrast $\delta \equiv (\rho - \bar{\rho})/\bar{\rho}$ deviates the Newtonian prediction

# Why dark energy in non-linear regime?

- DE was negligible at very early times
- DE becomes significant at later stage when non-linearities in cosmic structure are developed

Naturally DE affects the evolution of gravitational instability, so that its effects emerge more prominently at non-linear level

Thus of our interest are:

1. *relevance of GR*
2. *gauge issue in GR*
3. *effects of DE in non-linear regime of LSS*

# Newtonian theory

3 basic equations for density perturbation $\delta \equiv \delta\rho/\bar{\rho}$, peculiar velocity $\boldsymbol{u}$ and gravitational potential $\Phi$ with a *pressureless* fluid

$$\dot{\delta} + \frac{1}{a}\nabla\cdot\boldsymbol{u} = -\frac{1}{a}\nabla\cdot(\delta\boldsymbol{u}) \qquad \text{continuity eq}$$

$$\dot{\boldsymbol{u}} + H\boldsymbol{u} + \frac{1}{a}\nabla\Phi = -\frac{1}{a}(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} \qquad \text{Euler eq}$$

$$\frac{\Delta}{a^2}\Phi = 4\pi G\bar{\rho}\delta \qquad \text{Poisson eq}$$

Newtonian system is closed at 2nd order

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}\delta = -\frac{1}{a^2}\frac{d}{dt}\left[a\nabla\cdot(\delta\boldsymbol{u})\right] + \frac{1}{a^2}\nabla\cdot(\boldsymbol{u}\cdot\nabla\boldsymbol{u})$$

$\longrightarrow$ at linear order, $\delta_+ \propto a$ (growing) and $\delta_- \propto a^{-3/2}$ (decaying)

(Bernardeau et al. 2002)

# Basic non-linear equations

Based on the ADM metric

$$ds^2 = -N^2(dx^0)^2 + \gamma_{ij}\left(N^i dx^0 + dx^i\right)\left(N^j dx^0 + dx^j\right)$$

the fully non-linear equations are (Bardeen 1980)

$$R - \overline{K}^i{}_j\overline{K}^j{}_i + \frac{2}{3}K^2 - 16\pi GE = 0$$

$$\overline{K}^j{}_{i;j} - \frac{2}{3}K_{,i} = 8\pi GJ_i$$

$$\frac{K_{,0}}{N} - \frac{K_{,i}N^i}{N} + \frac{N^{;i}{}_{;i}}{N} - \overline{K}^i{}_j\overline{K}^j{}_i - \frac{1}{3}K^2 - 4\pi G(E+S) = 0$$

$$\frac{\overline{K}^i{}_{j,0}}{N} - \frac{\overline{K}^i{}_{j;k}N^k}{N} + \frac{\overline{K}_{jk}N^{i;k}}{N} - \frac{\overline{K}^i{}_k N^k{}_{;j}}{N} = K\overline{K}^i{}_j - \frac{1}{N}\left(N^{;i}{}_{;j} - \frac{\delta^i{}_j}{3}N^{;k}{}_{;k}\right) + \overline{R}^i{}_j - 8\pi G\overline{S}^i{}_j$$

$$\frac{E_{,0}}{N} - \frac{E_{,i}N^i}{N} - K\left(E + \frac{S}{3}\right) - \overline{K}^i{}_j\overline{S}^j{}_i + \frac{\left(N^2J^i\right)_{;i}}{N^2} = 0$$

$$\frac{J_{i,0}}{N} - \frac{J_{i;j}N^j}{N} - \frac{J_jN^j{}_{;i}}{N} - KJ_i + \frac{EN_{,i}}{N} + S^j{}_{i;j} + \frac{S^j{}_iN_{,j}}{N} = 0$$

Fluid quantities: $E \equiv n_\mu n_\nu T^{\mu\nu}$, $J_i \equiv -n_\mu T^\mu{}_i$, $S_{ij} \equiv T_{ij}$

# Setup and perturbation variables

We consider scalar metric pert in Einstein-de Sitter universe

$$N = 1 + \alpha \,, \quad N_i = a^2 \beta_{,i} \,, \quad \gamma_{ij} = a^2 \left[ (1 + 2\varphi) \delta_{ij} + \gamma_{,ij} \right]$$

The dynamical equations to be solved are:

$$\begin{aligned} &\text{Energy conservation eq} \rightarrow \text{Continuity eq} \\ &\text{Trace of the Einstein eq} \rightarrow \text{Euler eq} \end{aligned}$$

We identify the perturbation variables as

$$\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}} \quad \text{with} \quad \rho \equiv -T^0{}_0$$

$$\theta \equiv \frac{\nabla \cdot \boldsymbol{u}}{a} = -3H - K$$

# Strategy for non-linear perturbations

With the linear solution the same as the standard one

$$\delta_1(\boldsymbol{k},a) = D_1(a)\delta_1(\boldsymbol{k},a_0)$$

we expand $\delta = \delta_1 + \delta_2 + \cdots$ using symmetric kernels

$$\delta(\boldsymbol{k},a) = \sum_{n=1}^{\infty} D^n(a) \int \frac{d^3 d_1 \cdots d^3 q_n}{(2\pi)^{3(n-1)}} \delta^{(3)}\left(\boldsymbol{k} - \boldsymbol{q}_1 - \cdots \boldsymbol{q}_n\right) \times F_n(\boldsymbol{q}_1, \cdots \boldsymbol{q}_n)\delta_1(\boldsymbol{q}_1)\cdots\delta_1(\boldsymbol{q}_n)$$

Then correlation functions are

$$\left\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\right\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}+\boldsymbol{k}')P(k) \quad \text{with} \quad P = P_{11} + \underbrace{P_{22} + P_{13}}_{\text{1-loop}} + \cdots$$

$$\langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\delta(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_{123})B(k_1,k_2,k_3)$$
$$\text{with} \quad B = B_{112} + \underbrace{B_{222} + B_{123} + B_{114}}_{\text{1-loop}} + \cdots$$

# Comoving gauge

We set the gauge condition as

$$\gamma = 0 \quad \text{and} \quad T^0{}_i = 0$$

Kernels are found to be (Jeong, JG, Noh & Hwang 2011, Biern, JG & Jeong 2014)

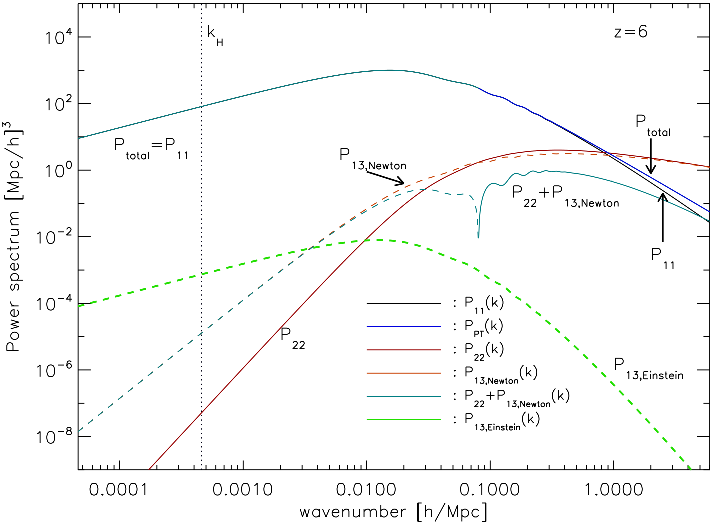
$$F_2 = \frac{5}{7} + \frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{2q_1q_2}\left(\frac{q_1}{q_2} + \frac{q_2}{q_1}\right) + \frac{2}{7}\left(\frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{q_1q_2}\right)^2$$

$$F_3 = F_{3\mathrm{N}} + F_{3\mathrm{GR}} \quad \text{where} \quad F_{3\mathrm{GR}} \propto k_H^2 \quad \text{with} \quad k_H \equiv aH$$

$$F_4 = F_{4\mathrm{N}} + (\cdots)k_H^2 + (\cdots)k_H^4$$

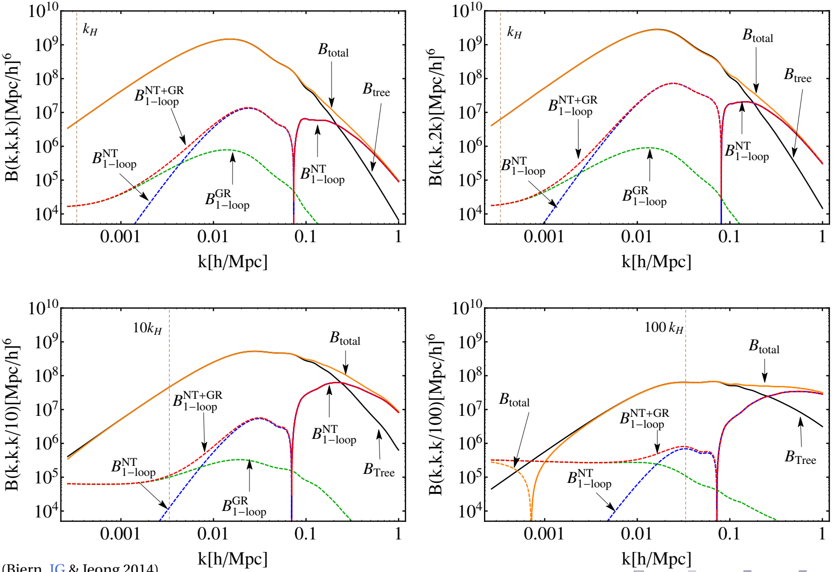
- Those w/o $\varphi$ are identical to the Newtonian kernels
- Newtonian kernels are the same as those found in the standard perturbation theory based on the Newtonian gravity
- GR contributions appear from 3rd order, prop to $k_H \equiv aH$

# Power spectrum with leading corrections in CG



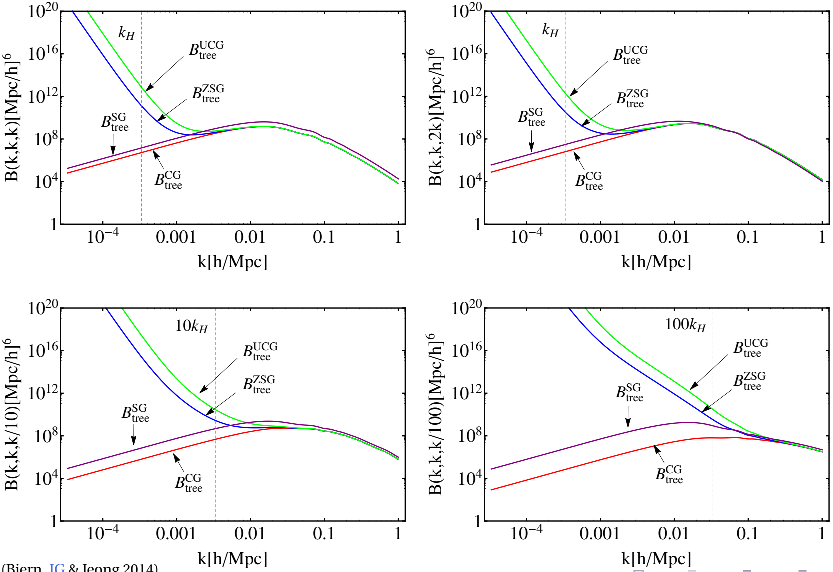

(Jeong, JG, Noh & Hwang 2011)

# Bispectrum with leading corrections in CG




(Biern, JG & Jeong 2014)

# Leading bispectrum in various gauges




(Biern, JG & Jeong 2014)

# Synchronous gauge

We set the gauge condition as

$$g_{00} = -1 \quad \text{and} \quad g_{0i} = 0$$

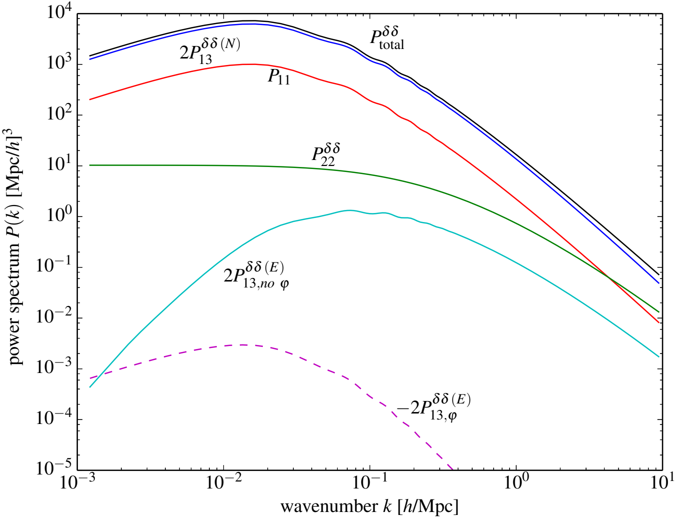
Kernels are found to be (Hwang, Noh, Jeong, JG & Biern 2014)

$$F_2 = \frac{5}{7} + \frac{2}{7}\frac{(\boldsymbol{q}_1 \cdot \boldsymbol{q}_2)^2}{q_1^2 q_2^2}$$

$$F_3 = F_{3\text{N}} + F_{3\text{GR},\varphi} + F_{3\text{GR,no}\ \varphi}$$

- Newtonian kernels are *different* from standard ones
- Some GR contributions are not from $\varphi$ but from non-linear coupling w/o $k_H$ (thus time independent)

# Power spectrum with leading corrections in SG

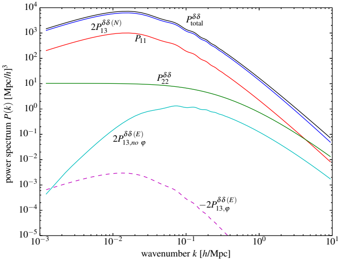



(Hwang, Noh, Jeong, JG & Biern 2014)

# Newtonian interpretation of CG and SG

The problem lies in the Newtonian contributions

$$\dot{\delta} + \frac{1}{a}(1+\delta)\nabla\cdot\boldsymbol{u} = 0\,, \quad \dot{\theta} + 2H\theta + 4\pi G\bar{\rho}\delta + \frac{1}{a^2}u^{i,j}u_{j,i} = (\text{NL terms})$$

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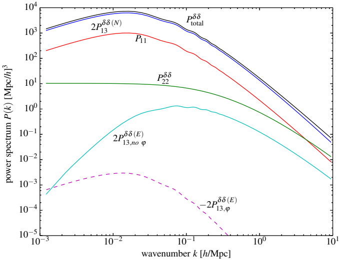
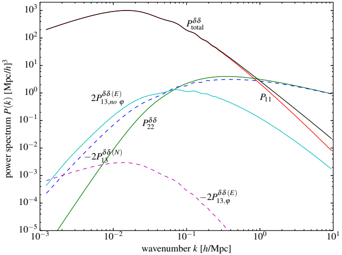
$$\Downarrow \quad \frac{d}{dt} \to \frac{d}{dt} + \frac{1}{a}\boldsymbol{u}\cdot\nabla \quad \text{transformation to convective derivative}$$

$$\dot{\delta} + \frac{1}{a}\nabla\cdot[(1+\delta)\boldsymbol{u}] = 0\,, \quad \dot{\theta} + 2H\theta + 4\pi G\bar{\rho}\delta + \frac{1}{a^2}\nabla\cdot[(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}] = (\text{NL terms})$$




(Hwang, Noh, Jeong, JG & Biern 2014)

# Putting dark energy on the table

Previous strategy is not complete

- $\Lambda$CDM power spectrum in EdS background
- Matter domination all the way

But we know the universe has been dominated by DE for a long time

$$\rho = \rho_m \longrightarrow \rho = \rho_m + \rho_{de} \quad \text{with} \quad p_{de} = w\rho_{de}$$

For simplicity

1. No DE perturbation: $\rho_{de} = \bar{\rho}_{de}$ (cf. Park, Hwang, Lee & Noh 2009)
2. Comoving gauge: $T^0{}_i = 0$

# Dark energy changes the game

DE provides different BG from both EdS and ΛCDM:

$$\mathcal{H}^2 = \frac{8\pi G}{3} a^2 \left(\bar{\rho}_m + \bar{\rho}_{de}\right) \quad \text{and} \quad \mathcal{H}' = -\frac{1}{2}\mathcal{H}^2 (1+3w)$$

DE permeates *all* order in perturbation: e.g. energy conservation

$$\delta' - \kappa(1-\lambda) = \text{(non-linear terms) where } \lambda \equiv (1+w)\left(1-\frac{1}{\Omega_m}\right)$$

Thus away from EdS ($\Omega_m = 1$) and ΛCDM ($w = -1$) the effects of general, dynamical DE are *manifest*: we use the parametrization

(Chevallier & Polarski 2001, Linder 2003)

$$w(a) = w_0 + (1-a) w_a$$

# Non-linear solutions with DE

Curvature perturbation is **not** conserved: from energy constraint

$$\varphi = -\frac{\mathcal{H}^2 f}{1-\lambda}\left[1+\frac{3}{2}(1-\lambda)\frac{\Omega_m}{f}\right]\Delta^{-1}\delta \neq \text{constant}$$

Thus $\delta$ receives a) curvature evolution effects from 3rd order and b) general, dynamical DE effects from BG and linear order:

$$\delta'' + \left(\mathcal{H} + \frac{\lambda'}{1-\lambda}\right)\delta' - \frac{3}{2}(1-\lambda)\mathcal{H}^2\Omega_m\delta = \underbrace{\mathcal{N}_N + \mathcal{N}_\varphi + \mathcal{N}_{\varphi'} + \mathcal{N}_\lambda}_{=\text{non-linear source terms}}$$

| | Newtonian | EdS | ΛCDM | DE |
|---|---|---|---|---|
| $\mathcal{N}_N$ | O | O | O | O |
| $\mathcal{N}_\varphi$ | X | O | O | O |
| $\mathcal{N}_{\varphi'}$ | X | X | X | O |
| $\mathcal{N}_\lambda$ | X | X | X | O |

# Relativistic kernels

2nd and 3rd order solutions are (Biern & JG 2015)

$$\delta_2(\boldsymbol{k},a) = D_1^2 \sum_{i=a}^{b} c_{2i}(a) \int \frac{d^3q_1 d^3q_2}{(2\pi)^3} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{q}_{12}) F_{2i}(\boldsymbol{q}_1,\boldsymbol{q}_2)\delta_1(\boldsymbol{q}_1)\delta_1(\boldsymbol{q}_2)$$

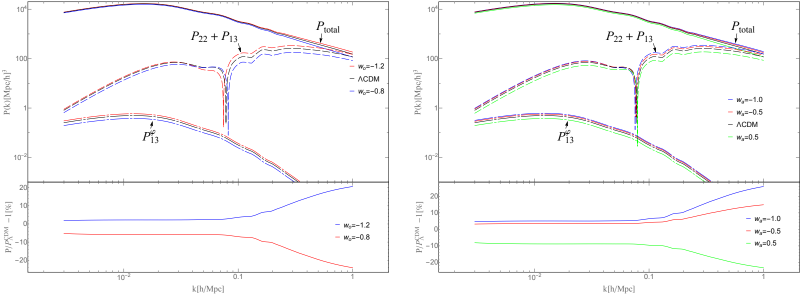
$$\delta_3(\boldsymbol{k},a) = D_1^3 \sum_{i=a}^{f} c_{3i}(a) \int \left[\cdots F_{3i} \cdots 3\ \delta_1' s\right] \qquad c_{ni} \equiv \frac{D_{ni}}{D_1^n}$$

$$+ D_1^3 \mathscr{H}^2 \sum_{i=a}^{b} c_{3i}^{\varphi}(a) \int \left[\cdots F_{3i}^{\varphi} \cdots 3\ \delta_1\text{'s}\right] \qquad c_{3i}^{\varphi} \equiv \frac{D_{3i}^{\varphi}}{D_1^3 \mathscr{H}^2}$$

In the EdS universe $c$'s are fixed as certain numbers ($c_{2a} = 3/7\ldots$) and (also in $\Lambda$CDM) $c_{ni}$ terms become purely Newtonian [Kamionkowski & Buchalter 1999 (2nd) and Takahashi 2008 (3rd)] and only $c_{3i}^{\varphi}$ terms remain relativistic

N. B. $\lambda$ is completely entangled and cannot be separated like $\varphi$

# One-loop corrected power spectrum: versus ΛCDM

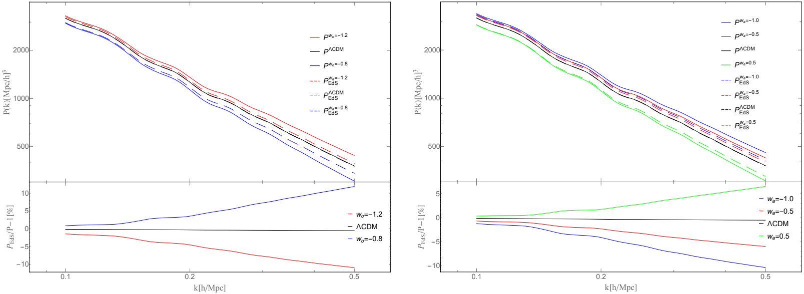



- Overall almost constant deviation on large scales ($k \lesssim 0.1 h$/Mpc)
- Deviation becomes significant on $k \gtrsim 0.1 h$/Mpc, close to baryon acoustic oscillations
- $w_0 > -1$ / $w_a > 0$ ($w_0 < -1$ / $w_a < 0$) give smaller (larger) $P(k)$

# One-loop corrected power spectrum: versus EdS

In Newtonian studies, usually EdS power spectrum is transferred to an arb model by replacing $a \to D_1(a)$:

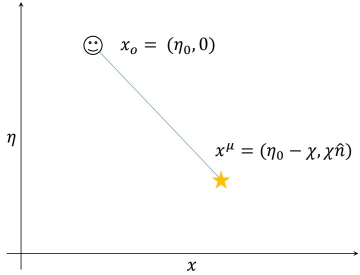
$$P(k,a) = D_1^2(a) P_{11}(k) + D_1^4(a) \left[ P_{22}(k) + P_{13}(k) \right]_{\rm EdS}$$




- For ΛCDM, only $\varphi$ drives difference so almost identical to EdS
- For general DE, the difference notably increases from $k \approx 0.1 h/\text{Mpc}$

# Observable galaxy number density

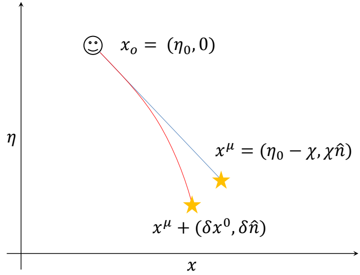
We observe as if photons come to us along a straight, unperturbed geodesic...



# Observable galaxy number density

We observe as if photons come to us along a straight, unperturbed geodesic... but in fact the path is distorted due to perturbations at the locations of the observer and the source, and in between

(Yoo et al. 2009, Bonvin & Durrer 2011, Bertacca, Maartens & Clarkson 2014, Yoo & Zaldarriaga 2014...)



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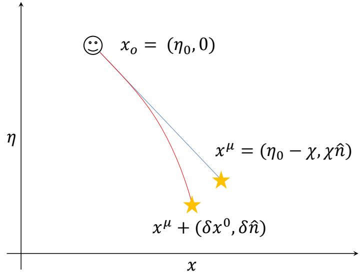
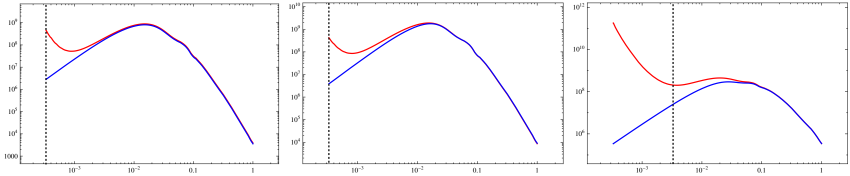



Observed number of galaxies $N$ contained in vol $\widetilde{V}$

$$N = \int_{\widetilde{V}} \sqrt{-g}\, n_g \varepsilon_{\mu\nu\rho\sigma} u^\mu \frac{\partial x^\nu}{\partial \tilde{x}^1} \frac{\partial x^\rho}{\partial \tilde{x}^2} \frac{\partial x^\sigma}{\partial \tilde{x}^3} d^3\tilde{x} \quad \rightarrow \quad \text{Galaxy field } \delta_g = (\cdots)$$

# Preliminary result

Galaxy bispectrum in different configurations (Biern, JG & Jeong; cf. Di Dio et al. 2014)




(Blue: Newtonian, red: Newtonian + GR contributions)

Work under progress!

# Conclusions

- As galaxy surveys become deeper and deeper, fully GR description is relevant
- Gauge dependence at non-linear order:
  - In CG the standard perturbation theory is reproduced
  - Pure GR corrections are heavily suppressed in almost all cases
  - Naively using SG leads to pathologies
  - Transformation by hands cures the problem
- With general dark energy:
  - Dark energy background greatly affects GR contributions
  - Notable difference of a few percent near BAO scales
  - Detectable signatures of judging $\Lambda$ or not
- Geodesic approach based on observable quantities should help