Deciphering the Majorana nature of sub-eV neutrinos by using their statistical property

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Contents

Introduction

Neutrinos have mass.

- Neutrinos are massless in SM, $m_\nu = 0$.
  All neutrinos are only left-handed (ν_L).

  $$\mathcal{L}^D_{\text{mass}} = -m_\nu(\overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R), \qquad m_\nu = \frac{Y_\nu \, \mathsf{v}}{\sqrt{2}},$$

  where Y_ν = Higgs-neutrino Yukawa coupling constant,
  and v = Higgs VEV.
  No way to generate mass without right-handed neutrinos (ν_R).

- But observations of **neutrino oscillation** imply that **neutrinos have mass**, $m_\nu \neq 0$.

  The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita (Super-Kamiokande) and Arthur B. McDonald (Sudbury Neutrino Observatory) “for the discovery of neutrino oscillations, which shows that neutrinos have mass”.

There are various suggestions as to how neutrinos can get mass.

- **Dirac mass:**
  - *Assumption:* ν_R exists.
  - *Lagrangian:*
  $$\mathcal{L}^D_{\mathrm{mass}} = -m^D_\nu \left(\overline{\nu_R}\,\nu_L + \overline{\nu_L}\,\nu_R\right).$$
  - *Disadvantage:* No reason for m^D_ν to be small.
- **Majorana mass:**
  - *Assumption:* neutrino $\equiv$ anti-neutrino.
  - *Lagrangian:*
  $$\mathcal{L}^M_{\mathrm{mass}} = \frac{1}{2} m^M_\nu \left(\overline{\nu^C_L}\,\nu_L + \overline{\nu_L}\,\nu^C_L\right).$$
  - *Disadvantage:* $\mathcal{L}^M_{\mathrm{mass}}$ is not invariant under $SU(2)_L \times U(1)_Y$ gauge group, so not allowed by SM.

Introduction

How to give neutrinos mass?

- **Dirac-Majorana mass:**
  - *Assumptions:* ν_R exists, and neutrino $\equiv$ anti-neutrino.
  - *Lagrangian:*

$$\mathscr{L}_{\text{mass}}^{D+M} = \tfrac{1}{2} m_\nu^L \left(\overline{\nu_L^C} \nu_L \right) + \tfrac{1}{2} m_\nu^R \left(\overline{\nu_R^C} \nu_R \right) - m_\nu^D \left(\overline{\nu_R} \nu_L \right) + \text{H.c.} = \tfrac{1}{2} \overline{N_L^C} M N_L + \text{H.c.},$$

where $N_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix}$ and $M = \begin{pmatrix} m_\nu^L & m_\nu^{D*} \\ m_\nu^D & m_\nu^R \end{pmatrix}$ is the mass matrix.
Note: Out of m_ν^L, m_ν^R and m_ν^D two are real and positive, but the remaining one is complex, in general. We choose m_ν^D to be complex.
Problem: ν_L and ν_R have no definite mass, due to Dirac mass m_ν^D.
Solution: Go to a basis in which mass matrix is diagonal. $N_L = U n_L$, with $n_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$ and $U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \equiv \mathrm{M}$.

$$\therefore \mathscr{L}_{\text{mass}}^{D+M} = \tfrac{1}{2} \left(m_1 \overline{\nu_1^C} \nu_1 + m_2 \overline{\nu_2^C} \nu_2 \right) + \text{H.c.},$$

with $\nu_k = \nu_{kL} + \nu_{kL}^C$ $(k = 1, 2)$ being Majorana neutrinos and

$$m_{2,1} = \tfrac{1}{2} \left(\left(m_\nu^L + m_\nu^R \right) \pm \sqrt{\left(m_\nu^L + m_\nu^R \right)^2 + 4 \left| m_\nu^D \right|^2} \right).$$

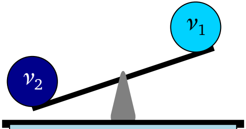
  - *Disadvantages:* (i) No explanation for small mass of neutrino, and (ii) one mass out of m_ν^D, m_ν^R, m_ν^L is complex, in general.

- **See-saw mechanism:** A simpler version of Dirac-Majorana mass, with a nice twist.
  - *Assumptions:* $m_\nu^L = 0$ and $m_\nu^D \ll m_\nu^R$.
  - *Lagrangian:* $\mathscr{L}_{\text{mass}}^{D+M} = \frac{1}{2} m_\nu^R \left(\overline{\nu_R^C} \nu_R\right) - m_\nu^D \left(\overline{\nu_R} \nu_L\right) + \text{H.c.} = \frac{1}{2} \overline{N_L^C} M N_L + \text{H.c.}$,

    where $N_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix}$ and $M = \begin{pmatrix} 0 & m_\nu^D \\ m_\nu^D & m_\nu^R \end{pmatrix}$ is the mass matrix.
  - *Mass eigenvalues:*

$$
\begin{aligned}
m_{2,1} &= \frac{1}{2}\left(m_\nu^R \pm \sqrt{\left(m_\nu^R\right)^2 + 4\left(m_\nu^D\right)^2}\right) \\
&\approx \frac{1}{2} m_\nu^R \left(1 \pm 1 \pm 2\left(\frac{m_\nu^D}{m_\nu^R}\right)^2\right).
\end{aligned}
$$

$$
\implies m_1 \approx -\frac{\left(m_\nu^D\right)^2}{m_\nu^R} \text{ and } m_2 \approx m_\nu^R.
$$

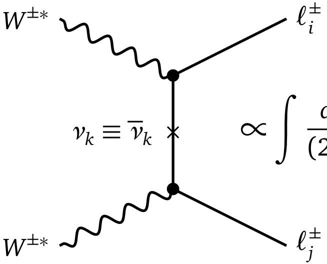


As ν_2 gets heavier, ν_1 become lighter.

  - *Advantage:* $m_1 \ll m_2$, so light neutrinos are possible.
  - *Challenges:*
    - To find the heavy ν_2 experimentally.
    - To prove that both the light ν_1 and heavy ν_2 are Majorana neutrinos.

Looking for Majorana neutrinos via $\Delta L = 2$ processes

- Neutrinos are the only *elementary fermions* known to us that *can* have Majorana nature.
- Majorana neutrinos: $\nu \equiv \overline{\nu}$.
- Majorana neutrinos violate lepton flavor number (L), they mediate $\Delta L = 2$ processes.



$$\propto \int \frac{d^4p}{(2\pi)^4} \sum_k U_{\ell_i k}\, U_{\ell_j k}\, \frac{m_k + \not{p}}{p^2 - m_k^2}$$

- $\Delta L = 2$ processes play crucial rule to probe Majorana nature of ν's.
  - neutrinoless double-beta ($0\nu\beta\beta$) decay
  - Rare meson decays with $\Delta L = 2$
  - Collider searches at LHC

Looking for Majorana neutrinos via $\Delta L = 2$ processes

- **Decay rate of any $\Delta L = 2$ process with final leptons $\ell_1^+ \ell_2^+$:**

$$\Gamma_{\Delta L=2} \propto \left| \sum_k U_{\ell_1 k} U_{\ell_2 k} \frac{m_k}{p^2 - m_k^2 + i m_k \Gamma_k} \right|^2 ,$$

where we have used the fact that $(1-\gamma^5)\not{p}(1-\gamma^5) = 0$.

- Light ν:

$$\Gamma_{\Delta L=2} \propto \left| \sum_k U_{\ell_1 k} U_{\ell_2 k} m_k \right|^2 = \left| m_{\ell_1 \ell_2} \right|^2 .$$

- Heavy ν:

$$\Gamma_{\Delta L=2} \propto \left| \sum_k \frac{U_{\ell_1 k} U_{\ell_2 k}}{m_k} \right|^2 .$$

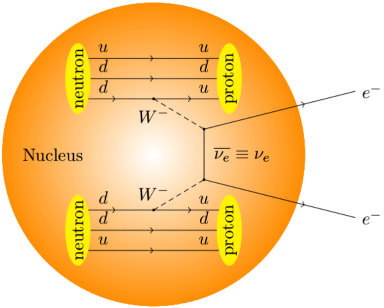
- Resonant ν:

$$\Gamma_{\Delta L=2} \propto \frac{\Gamma(N \to i)\, \Gamma(N \to f)}{m_N \Gamma_N} .$$

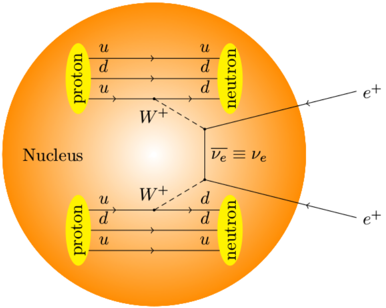
Looking for Majorana neutrinos via $\Delta L = 2$ processes

Neutrinoless double-beta ($0\nu\beta\beta$) decay

- Process:

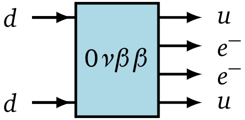


$^A_Z\mathcal{N} \longrightarrow ^A_{Z+2}\mathcal{N} + 2e^-$



$^A_Z\mathcal{N} \longrightarrow ^A_{Z-2}\mathcal{N} + 2e^+$

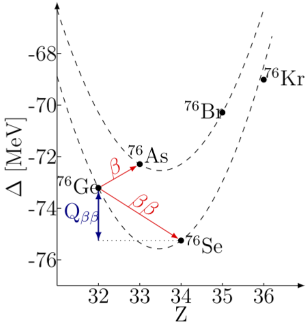
- Black-box diagram for $0\nu\beta\beta$

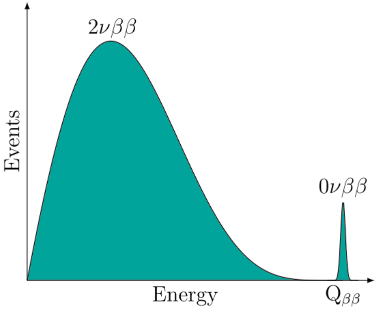


Looking for Majorana neutrinos via $\Delta L = 2$ processes

Neutrinoless double-beta ($0\nu\beta\beta$) decay

- Double-beta ($2\nu\beta\beta$) decay has been observed in 10 isotopes, ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te, ^{150}Nd, ^{238}U, with half-life $T_{1/2} \approx 10^{18} - 10^{24}$ years.





Giovanni Benato (for the GERDA collaboration), arXiv:1509.07792

- **$0\nu\beta\beta$ (forbidden in SM) is yet to be observed in any experiment.**

$$T_{1/2}^{0\nu}\left[{}^{76}\text{Ge}\right] \approx 1.9 \times 10^{25} \text{ years.}$$

Looking for Majorana neutrinos via $\Delta L = 2$ processes

Neutrinoless double-beta ($o\nu\beta\beta$) decay

- The half-life of a nucleus decaying via $0\nu\beta\beta$ is,

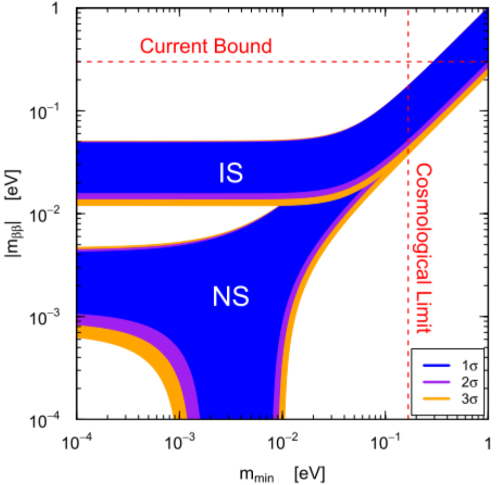
$$\left[T^{0\nu}_{1/2}\right]^{-1} = G_{0\nu}\ |M_{0\nu}|\ \left|m_{\beta\beta}\right|^2,$$

where

- $G_{0\nu}$ is phase space factor,
- $M_{0\nu}$ is the *nuclear matrix element*, (large theoretical uncertainty)
- $m_{\beta\beta}$ is *effective Majorana mass*. $m_{\beta\beta} = \sum_{k=1}^{3} U^2_{ek} m_k$ is complex, in general, and can be zero due to possible cancellations arising from phases in U_{ek}.

Looking for Majorana neutrinos via $\Delta L = 2$ processes

Neutrinoless double-beta ($0\nu\beta\beta$) decay



S. M. Bilenky and C. Giunti, arXiv:1203.5250 Mod. Phys. Lett. A **27**, 1230015 (2012)

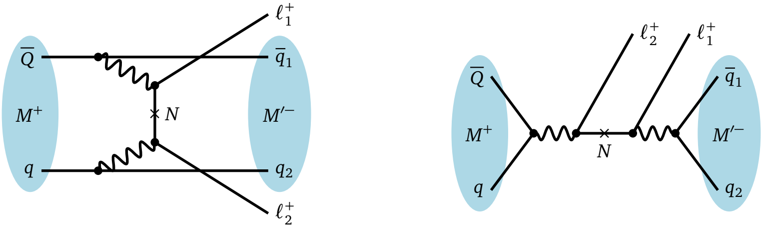
❖ **If $m_{\beta\beta}$ is vanishingly small, there will hardly be any neutrinoless double-beta decay events.**

Looking for Majorana neutrinos via $\Delta L = 2$ processes

Rare meson decays: $M^+ \to M'^- \ell_1^+ \ell_2^+$

- **Processes:** $M^+ \to M'^- \ell_1^+ \ell_2^+$,
  where $M = K, D, D_s, B, B_c$ and $M' = \pi, K, D, \ldots$

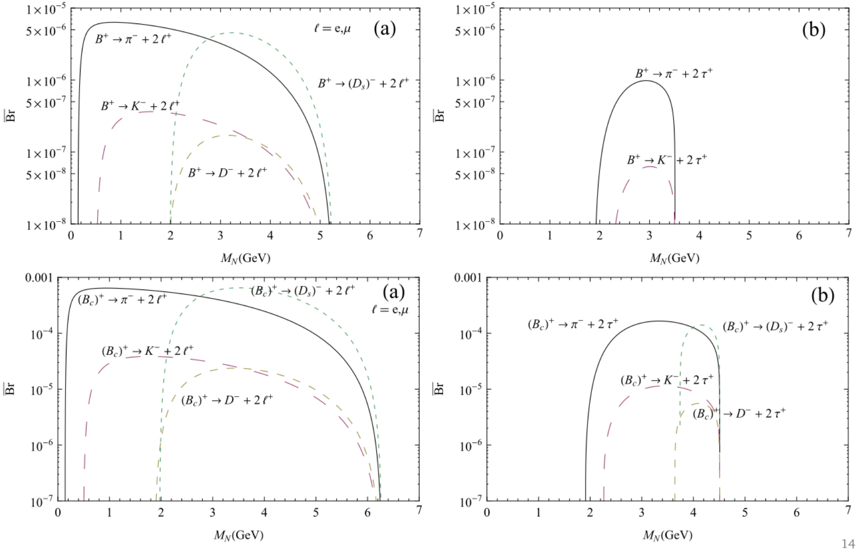
G. Cvetic, C.S. Kim, arXiv:1606.04140 (PRD **94**, 053001, 2016)
G. Cvetic, C. Dib, S. Kang, C. S. Kim,
arXiv:1005.4282 (PRD **82**, 053010, 2010)



- **No nuclear matrix element unlike** $0\nu\beta\beta$, but probes Majorana nature of massive neutrino(s) N.

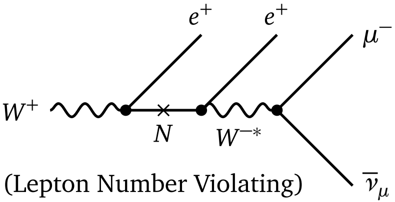
Looking for Majorana neutrinos via $\Delta L = 2$ processes

Rare meson decays: $M^+ \to M'^- \ell_1^+ \ell_2^+$

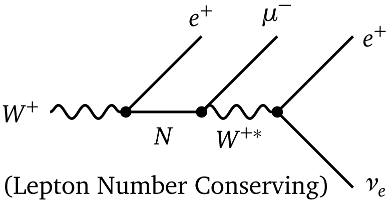


- **Processes:** $W^+ \to e^+e^+\mu^-\overline{\nu}_\mu$, $W^+ \to \mu^+\mu^+e^-\overline{\nu}_e$. Involves heavy neutrino N which can have Majorana nature as well.

C. Dib, C.S. Kim, arXiv:1509.05981 (PRD **92**, 093009, 2015);
C. Dib, C.S. Kim, K. Wang, J. Zhang, arXiv:1605.01123 (PRD **94**, 013005, 2016)



(Lepton Number Violating)



(Lepton Number Conserving)

- **Decay widths:**
  - LNV: $\Gamma\left(W^+ \to e^+e^+\mu^-\overline{\nu}_\mu\right) = |U_{Ne}|^4\,\hat{\Gamma}$,
  - LNC: $\Gamma\left(W^+ \to e^+e^+\mu^-\overline{\nu}_\mu\right) = \left|U_{Ne}U_{N\mu}\right|^2\hat{\Gamma}$,

where $\hat{\Gamma} = \dfrac{G_F^3 M_W^3}{12 \times 96\sqrt{2}\pi^4}\,\dfrac{m_N^5}{\Gamma_N}\left(1-\dfrac{m_N^2}{M_W^2}\right)^2\left(1-\dfrac{m_N^2}{2M_W^2}\right)$.

Synopsis: LHC Data Might Reveal Nature of Neutrinos

November 18, 2015

A long-standing question over whether the neutrino is its own antiparticle might be answered by looking at decays of *W* bosons.

As recognized by this year's Nobel Prize in physics, evidence now points to neutrinos having mass (see **7 October 2015 Focus** story). But this opens up new questions about why the neutrino mass is so much smaller than other particle masses. One solution is to assume that the neutrino is a different kind of particle—one that is its own antiparticle. A new theoretical study shows that observations of *W* boson decays at the Large Hadron Collider (LHC) in Geneva could potentially uncover the antiparticle nature of the neutrino.

Electrons, protons, and other fermions are Dirac particles, meaning they have a separate antiparticle with the same mass, but opposite charge. Neutrinos could be Dirac particles, but because they have no electric charge, they could also be Majorana particles, for which particle and antiparticle are the same thing. Such Majorana models are attractive because they offer a fairly natural explanation for the extremely small neutrino mass.

Experiments looking at extremely rare nuclear decays are trying to detect a possible Majorana or Dirac signature of the neutrino. To widen the search, Claudio Dib from Santa Maria University in Chile and Choong Sun Kim from Yonsei University in Korea propose looking at *W* boson decays. They considered decays that result in specific combinations of electrons, muons, and neutrinos. These decays have yet to be observed, but they are

Statistical Nature of Neutrinos

So far only the interaction properties of Majorana neutrinos have been exploited.

- Majorana neutrinos are a theorists favourite, because of their simplicity and the resulting elegance in theory, with exception of the nuclear matrix element in $0\nu\beta\beta$.

- In all major searches for Majorana neutrinos, for both active and heavy neutrino cases, we have exploited only their **mass dependent interaction property**, $\mathscr{L}_{\text{int}} = m_\nu \overline{\nu} \nu$.
  $\therefore m_\nu = 0 \implies$ no $0\nu\beta\beta$ decay or other $\Delta L = 2$ processes.
  Furthermore, for active neutrinos m_ν is very small, making the concerned processes **extremely rare**.

- Therefore, we want a better alternative to $0\nu\beta\beta$ decay or other $\Delta L = 2$ processes. These alternative processes,
  - Should not be a rare, i.e., when the neutrino mass is zero, the process must be allowed.
  - Must have a unique, experimentally observable signature for Majorana neutrinos.

- We shall explore the quantum statistical property of Majorana neutrinos which is independent of neutrino mass.

Statistical Nature of Neutrinos

The quantum statistical property of Majorana neutrinos does not depend upon their masses.

- If we have a neutrino and anti-neutrino pair (of the same flavor) in the final state of some process, the amplitude must be anti-symmetrized for Majorana neutrinos, but for Dirac case there is no such anti-symmetrization.

- This anti-symmetrization requirement for Majorana case arises because a Majorana neutrino and its anti-neutrino are *quantum mechanically indistinguishable*, i.e., they are identical fermions.

  Fermi-Dirac statistics for identical fermions

- This quantum statistical property does not depend on how heavy or light the Majorana neutrinos are.

- We shall consider processes whose final state contains $\nu_\ell \overline{\nu}_\ell$ (in addition to other particles) to explore the quantum statistical nature of Majorana neutrinos.

'Effective' Daitz plot method

We consider only such decay modes in which the 4-momentum of neutrino and anti-neutrino can be experimentally inferred.

Process: $X \to Y\left[\ell^+\ell^-\mathscr{Y}\right]\nu_\ell\overline{\nu}_\ell$ (an 'effective' three-body decay)

Conditions:

1. X is some suitable resonance.
2. Y is an 'effective' particle, which must always include $\ell^+\ell^-$, with some additional (not necessary) particle(s) $\mathscr{Y}$.
3. The 4-momenta of X and all particles in Y as well as those of some *intermediate resonances* must be experimentally measured such that 4-momenta of ν_ℓ and $\overline{\nu}_\ell$ are experimentally deducible.
4. $\nu_\ell\overline{\nu}_\ell$ do not arise from weak neutral current interaction in the process under consideration.

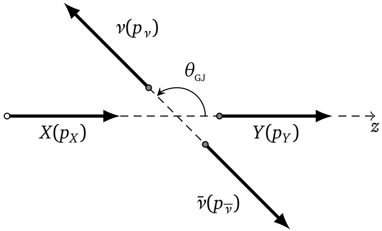
Some examples:

$$
\begin{aligned}
X\left[B^0\right] &\to \pi^+\left(\to \mu^+\nu_\mu\right)\mu^-\overline{\nu}_\mu && \equiv Y\left[\mu^+\mu^-\right]\nu_\mu\overline{\nu}_\mu,\\
X\left[B^0\right] &\to \pi^+\left(\to \mu^+\nu_\mu\right)\pi^-\left(\to \mu^-\overline{\nu}_\mu\right) && \equiv Y\left[\mu^+\mu^-\right]\nu_\mu\overline{\nu}_\mu,\\
X\left[B^+\right] &\to \bar{D}^0\left(\to K^+e^-\overline{\nu}_e\right)e^+\nu_e && \equiv Y\left[K^+e^+e^-\right]\nu_e\overline{\nu}_e.
\end{aligned}
$$

‘Effective’ Daitz plot method

We choose to work in a frame of reference in which exchange of ν and $\overline{\nu}$ is more elegant.

Gottfried-Jackson frame: $\vec{p}_\nu + \vec{p}_{\overline{\nu}} = \vec{0}$.



Invariant mass squares:

$$m^2_{\nu\overline{\nu}} = (p_\nu + p_{\overline{\nu}})^2 = (p_X - p_Y)^2,$$
$$m^2_{Y\nu} = (p_Y + p_\nu)^2 \equiv a - b\cos\theta_{\rm GJ},$$
$$m_{Y\overline{\nu}} = (p_Y + p_{\overline{\nu}})^2 \equiv a + b\cos\theta_{\rm GJ}.$$

where

$$a = \frac{1}{2}\left(m_X^2 + m_Y^2 + 2m_\nu^2 - s\right),$$
$$b = \frac{1}{2}\left(\sqrt{\lambda\left(m_X^2, m_Y^2, s\right)\left(1 - 4m_\nu^2/s\right)}\right),$$

with the Källén function $\lambda(x,y,z)$ defined as

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + yz + zx).$$

‘Effective’ Daitz plot method

Our tool for investigating the Majorana neutrinos is the ‘effective’ Dalitz plot.

- $m^2_{\nu\overline{\nu}} + m^2_{Y\nu} + m^2_{Y\overline{\nu}} = m^2_X + m^2_Y + 2m^2_\nu \equiv M^2$ (say). Since m^2_Y varies from event-to-event, M^2 does so also.

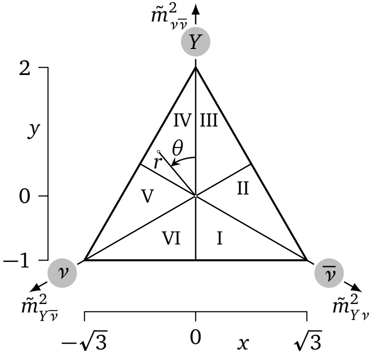
- Define new dimensionless ratios to take care of these event-to-event variations, $\tilde{m}^2_{\nu\overline{\nu}} \equiv m^2_{\nu\overline{\nu}}/M^2$, $\tilde{m}^2_{Y\nu} \equiv m^2_{Y\nu}/M^2$, $\tilde{m}^2_{Y\overline{\nu}} \equiv m^2_{Y\overline{\nu}}/M^2$, such that $\tilde{m}^2_{\nu\overline{\nu}} + \tilde{m}^2_{Y\nu} + \tilde{m}^2_{Y\overline{\nu}} = 1$.

- We can always construct a ternary plot (which along with event points we shall refer to as the ‘effective’ Dalitz plot) using $(\tilde{m}^2_{Y\nu}, \tilde{m}^2_{Y\overline{\nu}}, \tilde{m}^2_{\nu\overline{\nu}})$ as Cartesian coordinates.

‘Effective’ Daitz plot method

Our tool for investigating the Majorana neutrinos is the ‘effective’ Dalitz plot.

Any point inside the ternary plot can be described by either polar coordinates (r, θ) or rectangular coordinates (x, y).



$$\tilde{m}^2_{\nu\overline{\nu}} = \frac{1}{3}(1 + r\ \cos\theta) = \frac{1}{3}(1 + y),$$

$$\tilde{m}^2_{Y\nu} = \frac{1}{3}\left(1 + r\ \cos\left(\frac{2\pi}{3} + \theta\right)\right) = \frac{1}{6}\left(2 + \sqrt{3}x - y\right),$$

$$\tilde{m}^2_{Y\overline{\nu}} = \frac{1}{3}\left(1 + r\ \cos\left(\frac{2\pi}{3} - \theta\right)\right) = \frac{1}{6}\left(2 - \sqrt{3}x - y\right).$$

$$\tilde{m}^2_{Y\nu} \longleftrightarrow \tilde{m}^2_{Y\overline{\nu}} \equiv \theta_{\text{GJ}} \longleftrightarrow \pi - \theta_{\text{GJ}} \equiv \theta \longleftrightarrow -\theta.$$

‘Effective’ Daitz plot method

We analyse the distribution of events in the ‘effective’ Dalitz plot to distinguish Dirac and Majorana neutrinos.

- The pattern of distribution of events in the ‘effective’ Dalitz plot is a consequence of *dynamics*.

- The dynamics is encoded in the *transition amplitude*.

- The amplitude for all the processes under our consideration, should be *anti-symmetrized for Majorana neutrinos*, while for Dirac case there is no such anti-symmetrization.

- The distribution of events should be completely symmetric under exchange of ν and $\overline{\nu}$ for Majorana neutrinos. For Dirac neutrinos the distribution must have some asymmetry under the above exchange.

'Effective' Daitz plot method

Let us analyse an example process: $X\left[B^0\right] \to \pi^+\left(\to \mu^+ \nu_\mu\right) \pi^-\left(\to \mu^- \overline{\nu}_\mu\right) \equiv Y\left[\mu^+\mu^-\right] \nu_\mu \overline{\nu}_\mu$.

Amplitude: Using Fierz rearrangement theorem,

- Dirac neutrinos:

$$\begin{aligned}\mathscr{M}_D \propto (p_- + p_{\overline{\nu}})_\alpha \; (p_+ + p_\nu)_\beta \; & \left[\overline{\psi}_{\mu^-}(p_-)\,\gamma^\alpha\left(1-\gamma^5\right)\psi_{\mu^+}(p_+)\right] \\ & \times \left[\overline{\psi}_\nu(p_\nu)\,\gamma^\beta\left(1-\gamma^5\right)\psi_{\overline{\nu}}(p_{\overline{\nu}})\right],\end{aligned}$$

- Majorana neutrinos:

$$\begin{aligned}\mathscr{M}_M \propto -\Big(& (p_- + p_{\overline{\nu}})_\alpha \; (p_+ + p_\nu)_\beta + (p_- + p_\nu)_\alpha \; (p_+ + p_{\overline{\nu}})_\beta \Big) \\ & \times \left[\overline{\psi}_{\mu^-}(p_-)\,\gamma^\alpha\left(1-\gamma^5\right)\psi_{\mu^+}(p_+)\right]\left[\overline{\psi}_\nu(p_\nu)\,\gamma^\beta\gamma^5\psi_{\overline{\nu}}(p_{\overline{\nu}})\right],\end{aligned}$$

‘Effective’ Daitz plot method

Let us analyse an example process: $X\left[B^0\right] \to \pi^+\left(\to \mu^+ \nu_\mu\right)\pi^-\left(\to \mu^- \overline{\nu}_\mu\right) \equiv Y\left[\mu^+\mu^-\right]\nu_\mu\overline{\nu}_\mu$.

Squaring the modulus of amplitude and summing over final spins, with the simplifying assumption that $m_\mu = 0 = m_\nu$, we get,

- Dirac neutrinos:

$$\left\langle \left|\mathscr{M}_D\right|^2 \right\rangle \propto 256\,(p_- \cdot p_{\overline{\nu}})\,(p_\nu \cdot p_+)\,(p_{\overline{\nu}} \cdot p_+)^2\,,$$

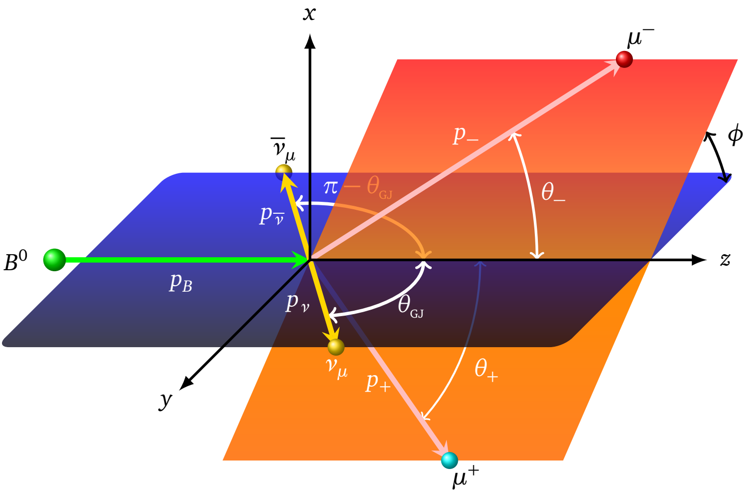
- Majorana neutrinos:

$$\begin{aligned}\left\langle \left|\mathscr{M}_M\right|^2 \right\rangle \propto 128\,(p_\nu \cdot p_+)\,(p_{\overline{\nu}} \cdot p_+)\Big(&-(p_- \cdot p_+)\,(p_\nu \cdot p_{\overline{\nu}}) \\ &+ (p_- \cdot (p_\nu + p_{\overline{\nu}}))\,(p_+ \cdot (p_\nu + p_{\overline{\nu}}))\Big).\end{aligned}$$

‘Effective’ Daitz plot method

Let us analyse an example process: $X\left[B^0\right] \to \pi^+\left(\to \mu^+ \nu_\mu\right) \pi^-\left(\to \mu^- \overline{\nu}_\mu\right) \equiv Y\left[\mu^+ \mu^-\right] \nu_\mu \overline{\nu}_\mu$.

Kinematics:



‘Effective’ Daitz plot method

Let us analyse an example process: $X\left[B^0\right] \to \pi^+\left(\to \mu^+\nu_\mu\right)\pi^-\left(\to \mu^-\overline{\nu}_\mu\right) \equiv Y\left[\mu^+\mu^-\right]\nu_\mu\overline{\nu}_\mu$.

Integrate the amplitude modulus square over ϕ and get,

- ❖ Dirac neutrinos:

$$\left\langle \left|\mathscr{M}_D\right|^2 \right\rangle \propto 32\pi m_\pi^4 \Big(\left(2E_+^2 - 3\mathsf{p}'^2\right)\mathsf{p}^2\cos(2\theta_{\rm GJ}) + 8\left(\mathsf{p}\mathsf{E}E_+\sqrt{E_+^2-\mathsf{p}'^2}\right)\cos\theta_{\rm GJ} + \left(2E_+^2-\mathsf{p}'^2\right)\mathsf{p}^2 + 4\mathsf{E}^2E_+^2\Big).$$

- ❖ Majorana neutrinos:

$$\left\langle \left|\mathscr{M}_M\right|^2 \right\rangle \propto 32 m_\pi^4 \left(\mathsf{E}^2\left(m_B^2 + 4\mathsf{E}^2 - 2\mathsf{E}E_B + 4E_+E_-\right)\right).$$

Here all quantities are in the Gottfried-Jackson frame, and

- $E_B, E_\pm, \mathsf{E}$ are the energies of B meson, $\mu^\pm$, ν_μ (or $\overline{\nu}_\mu$) respectively,
- p is the magnitude of the 3-momentum of ν_μ (or $\overline{\nu}_\mu$),
- p' is the magnitude of the projection of 3-momentum of $\mu^\pm$ on the xy-plane.

The term in red is proportional to $\cos\theta_{\rm GJ}$ and renders $\left\langle \left|\mathscr{M}_D\right|^2 \right\rangle$ *not symmetric* under $\nu \longleftrightarrow \overline{\nu}$ exchange which gets reflected in the distribution of events in the ‘effective’ Dalitz plot.

‘Effective’ Daitz plot method

Let us analyse an example process: $X\left[B^0\right] \to \pi^+ \left(\to \mu^+ \nu_\mu\right) \pi^- \left(\to \mu^- \overline{\nu}_\mu\right) \equiv Y\left[\mu^+ \mu^-\right] \nu_\mu \overline{\nu}_\mu$.

Signature of Majorana neutrinos: The distribution of events in the ‘effective’ Dalitz plot is completely symmetric under $\tilde{m}^2_{Y\nu} \longleftrightarrow \tilde{m}^2_{Y\overline{\nu}}$ for Majorana neutrinos. Dirac neutrinos lead to an asymmetry in the ‘effective’ Dalitz plot under the same exchange.

‘Effective’ Daitz plot method

The distribution of events in the ‘effective’ Dalitz plot can be described by a Fourier decomposition.

- We can construct the full ‘effective’ Dalitz plot, $0 \leqslant \theta \leqslant 2\pi$.
- The distribution of events inside it has no discontinuity, or infiniteness.
- Let $\mathscr{D}(r,\theta)$ denote the distribution of events inside the ‘effective’ Dalitz plot. Then,
  - $\mathscr{D}_D(r,\theta) = \sum_{n=0}^{\infty} \left(S_n^D(r)\ \sin(n\theta) + C_n^D(r)\ \cos(n\theta) \right)$ (Dirac neutrinos)
  - $\mathscr{D}_M(r,\theta) = \sum_{n=0}^{\infty} C_n^M(r)\ \cos(n\theta)$ (Majorana neutrinos)

  where $S_n^D(r)$ and $C_n^{D,M}(r)$ are the Fourier coefficients which are some functions of masses and energies of the particles involved.

‘Effective’ Daitz plot method

The Dirac and Majorana neutrinos leave two distinct signatures in the ‘effective’ Dalitz plot.

- $\int dr\, \mathscr{D}_M(r,\theta) = \int dr\, \mathscr{D}_M(r,-\theta),$ (Majorana neutrinos)
- $\int dr\, \mathscr{D}_D(r,\theta) \neq \int dr\, \mathscr{D}_D(r,-\theta),$ (Dirac neutrinos)

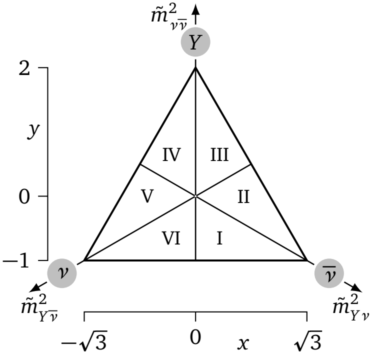
where we have carried out integrations radially, i.e. we add all the events inside the ‘effective’ Dalitz plot along the radial direction at any chosen polar angle.

It must be emphasized that this distinction between Dirac and Majorana neutrinos is always present in our ‘effective’ Dalitz plot *irrespective of neutrino mass*.

‘Effective’ Daitz plot method

The signature of Majorana neutrinos can be quantified in terms of some easily observable asymmetries.

❖ **Sextant asymmetry:**



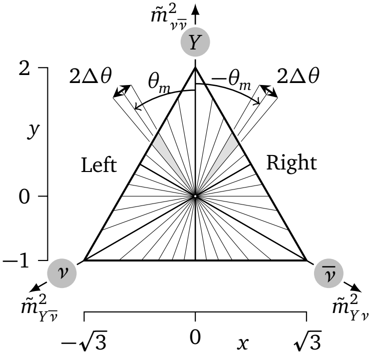
$$\mathscr{A} = \left|\frac{N_I - N_{VI}}{N_I + N_{VI}}\right| + \left|\frac{N_{II} - N_V}{N_{II} + N_V}\right| + \left|\frac{N_{III} - N_{IV}}{N_{III} + N_{IV}}\right|,$$

where N_i denotes the number of events in the ith sextant.

‘Effective’ Daitz plot method

The signature of Majorana neutrinos can be quantified in terms of some easily observable asymmetries.

❖ **Binned asymmetry:**



$$\mathscr{A}' = \sum_{\theta_m} \left| \frac{N(\theta_m) - N(-\theta_m)}{N(\theta_m) + N(-\theta_m)} \right|,$$

where $N(\theta_m)$ is the number of events in an angular bin $\theta_m \pm \Delta\theta$.

‘Effective’ Daitz plot method

There exist a plethora of processes which can be looked at using our approach.

X	$\rightarrow$	intermediate resonances	$\rightarrow$	final state $(Y)\,\nu_\ell\,\overline{\nu}_\ell$
B^0		$D^-\ell^+\nu_\ell$		$\left(K^0\ell^+\ell^-\right)\nu_\ell\,\overline{\nu}_\ell$
B^+		$\bar{D}^0\ell^+\nu_\ell$		$(K^-\ell^+\ell^-)\,\nu_\ell\,\overline{\nu}_\ell$
B^0, D^0, K^0		$\pi^\pm\ell^\mp\overset{(-)}{\nu}_\ell$		$(\ell^+\ell^-)\,\nu_\ell\,\overline{\nu}_\ell$
		$\pi^+\pi^-$		$(\ell^+\ell^-)\,\nu_\ell\,\overline{\nu}_\ell$
$\overline{B}^0, D^0$		π^+K^-		$(\ell^+\ell^-)\,\nu_\ell\,\overline{\nu}_\ell$
K_S^0		$\pi^+\pi^-\gamma$		$(\ell^+\ell^-\gamma)\,\nu_\ell\,\overline{\nu}_\ell$
B^0, D^0, $K_L^0, J/\psi(1S)$		$\pi^+\pi^-\pi^0$		$(\ell^+\ell^-\pi^0)\,\nu_\ell\,\overline{\nu}_\ell$

'Effective' Daitz plot method

There exist a plethora of processes which can be looked at using our approach.

X	$\to$	intermediate resonances	$\to$	final state $(Y)\nu_\ell\overline{\nu}_\ell$
D^0		$\pi^+\pi^-K_S^0$		$(\ell^+\ell^-K_S^0)\nu_\ell\overline{\nu}_\ell$
		$\pi^+K^-\pi^0$		$(\ell^+\ell^-\pi^0)\nu_\ell\overline{\nu}_\ell$
		$K^+K^-K_S^0$		$(\ell^+\ell^-K_S^0)\nu_\ell\overline{\nu}_\ell$
		$\pi^+\pi^-2\pi^0$		$(\ell^+\ell^-2\pi^0)\nu_\ell\overline{\nu}_\ell$
		$K^-\ell^+\nu_\ell$		$(\ell^+\ell^-)\,\nu_\ell\overline{\nu}_\ell$
D^+		$\bar{K}^0\ell^+\nu_\ell$		$(\pi^+\ell^-\ell^+)\,\nu_\ell\overline{\nu}_\ell$
$J/\psi(1S)$		$\pi^+\pi^-\omega$		$(\ell^+\ell^-\omega)\nu_\ell\overline{\nu}_\ell$
		$\pi^+\pi^-\eta$		$(\ell^+\ell^-\eta)\nu_\ell\overline{\nu}_\ell$
		$\pi^+\pi^-\phi$		$(\ell^+\ell^-\phi)\nu_\ell\overline{\nu}_\ell$
		$\pi^+\pi^-\omega\pi^0$		$(\ell^+\ell^-\omega\pi^0)\nu_\ell\overline{\nu}_\ell$
$\Upsilon(2S)$		$\pi^+\pi^-\Upsilon(1S)$		$(\ell^+\ell^-\Upsilon(1S))\nu_\ell\overline{\nu}_\ell$

Conclusion

- Processes are *not rare* for our case. These are not dependent on neutrino mass, and we are using the statistical property of neutrinos.

- Majorana and Dirac neutrinos have completely distinct signatures, which survive even when one considers neutrinos to be massless.

- The signatures are quantifiable by easily observable asymmetries defined on 'effective' Dalitz plots.

- For $m_\nu \to 0$, $\Delta L = 2$ processes $\to 0$, but our asymmetries $\not\to 0$.

Conclusion

By using our methodology of considering ‘effective’ Dalitz plots for suitably well choosen processes one can look for the Majorana nature of active neutrinos.

Thank You