Softly broken A_4 symmetry for nearly degenerate neutrino masses

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1. Introduction

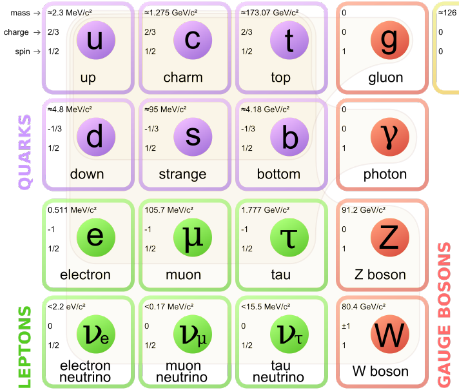


Figure: The Standard Model

Some deficiencies of the SM

- The origin of mass
- The strong CP problem
- Neutrino oscillations
- Matter-antimatter asymmetry
- The natrue of DM and DE

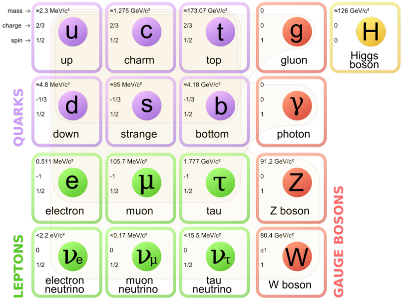


Figure: The Standard Model

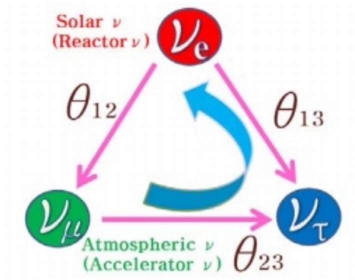


Figure: Neutrino oscillations

- Neutrinos should be massive!
- We need an appropriate theory to fill the gap.

} We need BSM!

1. Introduction

Quantity	Three-neutrino mixing parameters from pdg
$\Delta m^2_{sun} = \Delta m^2_{21} (10^{-5}\,\mathrm{eV}^2)$	7.53 ± 0.18
$\lvert\Delta m^2_{atm}\rvert = \lvert\Delta m^2_{32} (10^{-3}\,\mathrm{eV}^2)\rvert$	2.42 ± 0.06
$\sin^2\theta_{12}$	0.304 ± 0.014
$\sin^2 2\theta_{12}$	0.846 ± 0.021
$\sin^2\theta_{23}$	$0.514^{+0.055}_{-0.056}$
$\sin^2 2\theta_{23}$	$0.999^{+0.001}_{-0.018}$
$\sin^2\theta_{13}$	0.0219 ± 0.0012
$\sin^2 2\theta_{13}$	0.085 ± 0.005
δ_{CP}	$\pm\pi/2$

Table: Neutrino oscillation data

Oscillation experiments do NOT provide information about

- absolute neutrino mass scale
- Dirac/Majorana nature of neutrinos

1. Introduction

We don't know absolute ν mass scale. So we have two possible scenarios based on the experimental results.

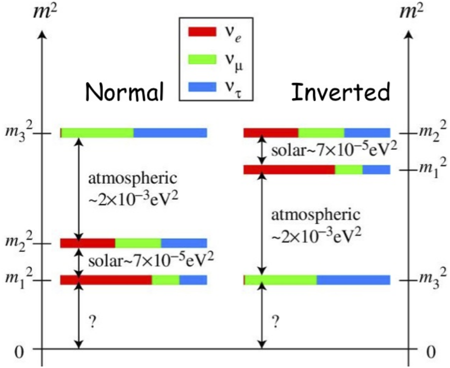


Figure: Normal and Inverted hierarchy

- $|\Delta m^2_{32}| \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2$
- $\Delta m^2_{21} \simeq 7.5 \times 10^{-5}\,\mathrm{eV}^2$

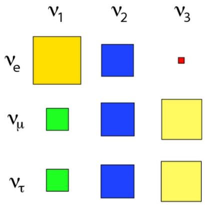
From this graph, we can naturally come up with neutrino mixing.

1. Introduction

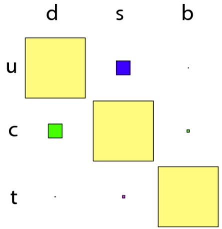
Neutrino mixing is important because it could provide new clues for the understanding of the flavor problem.

Neutrino mixing pattern is completely different that of quark mixing.

Neutrino Mixing



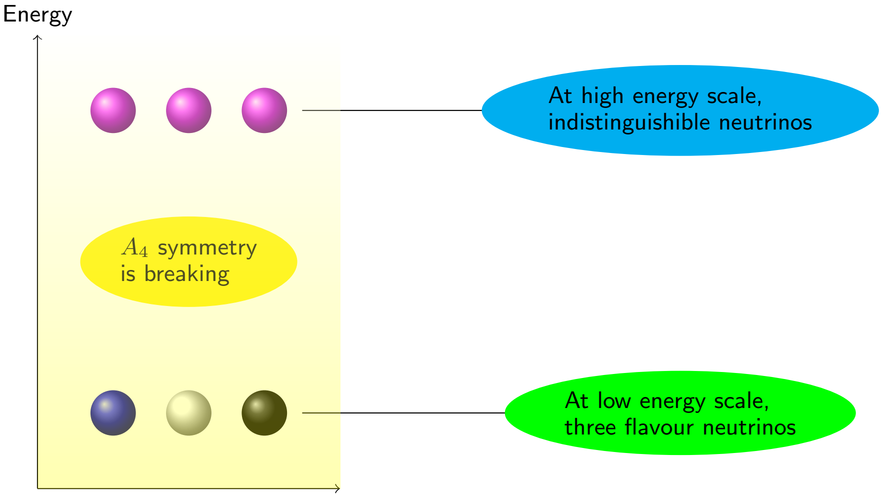
Quark Mixing



From neutrino mixing, we could expect a specific symmetry!

1. Introduction

Then you can ask me why you take a specific symmetry as for neutrino research.



1. Introduction

In order to read a specific symmetry from neutrino mixing, lots of special groups have been studied by neutrino theorists.

Group	d	Irr. Repr.'s	Presentation
$D_3 \sim S_3$	6	$1, 1', 2$	$A^3 = B^2 = (AB)^2 = 1$
D_4	8	$1_1, \cdot, 1_4, 2$	$A^4 = B^2 = (AB)^2 = 1$
D_7	14	$1, 1', 2, 2', 2''$	$A^7 = B^2 = (AB)^3 = 1$
A_4	12	$1, 1', 1'', 3$	$A^3 = B^2 = (AB)^3 = 1$
$A_5 \sim PSL_2(5)$	60	$1, 3, 3', 4, 5$	$A^3 = B^2 = (BA)^5 = 1$
T'	24	$1, 1', 1'', 2, 2', 2'', 3$	$A^3 = (AB)^3 = R^2 = 1, B^2 = R$
S_4	24	$1, 1', 2, 3, 3'$	$BM \ : \ A^4 = B^2 = (AB)^3 = 1$ $TB \ : \ A^3 = B^4 = (BA^2)^2 = 1$
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	$1_1, \cdot, 1_9, 3, \bar{3}$	
$PSL_2(7)$	168	$1, 3, \bar{3}, 6, 7, 8$	$A^3 = B^2 = (BA)^7 = (B^{-1}A^{-1}BA)^4 = 1$
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \bar{1}', 3, \bar{3}$	$A^7 = B^3 = 1, AB = BA^4$

Table: Some small discrete groups used for model building.

- Tri-Bimaximal mixing : mixing equally ν_e with ν_μ, and ν_τ
- Bimaximal mixing : mixing equally ν_μ with ν_τ

The charged lepton masses are certainly not degenerate, so whatever symmetry we use to maintain the neutrino mass degeneracy must be broken.

Figure: Each lepton mass

To implement this idea in a renormalizable field theory, the symmetry in question should be broken only spontaneously and by explicit soft terms(if it is not a gauge symmetry).

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2. Discrete symmetry A_4

The A_4 symmetry is group of the even permutations of S_4.

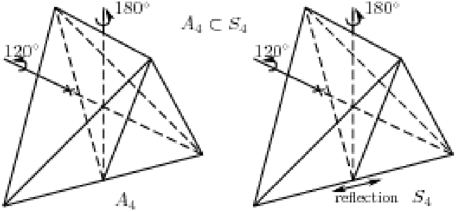

Figure: The A_4 symmetry

The A_4 symmetry

- has 12 elements
- are divided into 4 classes, with the number of elements 1, 4, 4, 3, respectively
- has 4 irreducible representations, with dimensions n_i, such that $\sum_i n_i^2 = 12$

2. Discrete symmetry A_4

There is only one solution corresponding to $\sum_i n_i^2 = 12$.

$$n_1 = n_2 = n_3 = 1, \text{ and } n_4 = 3,$$

From now on, I call the irreducible representations as below

- $\mathbf{1}$: having dimension $n_1 = 1$
- $\mathbf{1}'$: having dimension $n_2 = 1$
- $\mathbf{1}''$: having dimension $n_3 = 1$
- $\mathbf{3}$: having dimension $n_4 = 3$

2. Discrete symmetry A_4

Let me go over classes of A_4 symmetry. First of all, all of the A_4 elements are written by products of the generator, s and t, which satisfy

$$s^2 = t^3 = (st)^3 = e$$

- C_1 : $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$,
- C_2 : $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$,
- C_3 : $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$,
- C_4 : $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

2. Discrete symmetry A_4

Now, I know both irreducible representations for A_4 symmetry and the classes of the symmetry A_4.

Then I can write down the character table for the A_4 symmetry.

A_4	$1C^1(1)$	$3C^2(s)$	$4C^3(t)$	$4C^3(t^2)$
$\chi_i^{[1]}$	1	1	1	1
$\chi_i^{[1']}$	1	1	ω	ω^2
$\chi_i^{[1'']}$	1	1	ω^2	ω
$\chi_i^{[3]}$	3	-1	0	0

Table: The character table for the A_4 symmetry

It is possible to decompose the Kronecker products of two multiplets now.

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3} \oplus \mathbf{3}$$

Please keep in mind that this decomposition of the Kronecker products does not depend on the choice of basis.

2. Discrete symmetry A_4

Until now, I have used the bases for the generators s and t on the representation $\mathbf{3}$.

- $s = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$
- $t = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

And then, I will consider another basis for the A_4 symmetry.

- $a = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$
- $b = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$

These bases are transformed by the following unitary transformation U_ω as

$$U_\omega = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega, \end{pmatrix}$$

and the elements a and b are written as

$$a = U_\omega^\dagger s U_\omega, \qquad b = U_\omega^\dagger t U_\omega$$

2. Discrete symmetry A_4

Then, let me compare decomposition of the Kronecker products of two triplets by one basis with it by another basis.

The Kronecker products of two triplets by one basis

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} = (a_1b_1 + a_2b_2 + a_3b_3)_{\mathbf{1}} \oplus (a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3)_{\mathbf{1'}} \oplus$$

$$(a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3)_{\mathbf{1''}} \oplus \begin{pmatrix} a_2b_3 + b_3a_2 \\ a_3b_1 + b_1a_3 \\ a_1b_2 + b_2a_1 \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_2b_3 - b_3a_2 \\ a_3b_1 - b_1a_3 \\ a_1b_2 - b_2a_1 \end{pmatrix}_{\mathbf{3}}$$

The Kronecker products of two triplets by another basis

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} = (a_1b_1 + a_2b_3 + a_3b_2)_{\mathbf{1}} \oplus (a_3b_3 + a_1b_2 + a_2b_1)_{\mathbf{1'}} \oplus$$

$$(a_2b_2 + a_1b_3 + a_3b_1)_{\mathbf{1''}} \oplus \frac{1}{3}\begin{pmatrix} 2a_1b_1 - a_2b_3 - a_3b_2 \\ 2a_3b_3 - a_1b_2 - a_2b_1 \\ 2a_2b_2 - a_1b_3 - a_3b_1 \end{pmatrix}_{\mathbf{3}} \oplus \frac{1}{2}\begin{pmatrix} a_2b_3 - a_3b_2 \\ a_1b_2 - a_2b_1 \\ a_1b_3 - a_3b_1 \end{pmatrix}_{\mathbf{3}}$$

Before to go on

One assigns **leptons** to the **four inequivalent representations of A_4**:

$$\underbrace{\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L \quad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L \quad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L}_{\mathbf{3}} \qquad \Bigg| \qquad \underbrace{e_R}_{\mathbf{1}} \quad \underbrace{\mu_R}_{\mathbf{1}''} \quad \underbrace{\tau_R}_{\mathbf{1}'}$$

Here we consider a **see-saw realization**, so we also introduce **right-handed neutrino fields ν_R** transforming as a triplet of A_4

$$\underbrace{\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L \quad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L \quad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L}_{\substack{\nu_{e_R} \quad \nu_{\mu_R} \quad \nu_{\tau_R} \\ \mathbf{3}}} \qquad \Bigg| \qquad \underbrace{e_R}_{\mathbf{1}} \quad \underbrace{\mu_R}_{\mathbf{1}''} \quad \underbrace{\tau_R}_{\mathbf{1}'}$$

We adopt a **supersymmetric (SUSY)** also to make contact with **Grand Unification**.

$\rightarrow$ Flavor symmetries are supposed to act near the GUT scale.

2. Discrete symmetry A_4

The reason that we choose A_4 for discussing degenerate neutrino masses is that

1. it is simplest.
2. it is ideal for having degenerate Dirac neutrino masses while allowing arbitrary charged-lepton masses.

In contrast,

- The S_3 discrete symmetry has one **2** and two **1**.
- The S_4 discrete symmetry has two **3** and one **2** and two **1**.
- If continuous groups are considered, then $SO(3)$ has a three dimensional representation and may be used as well.

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3. Model of nearly degenerate neutrino masses

Under A_4 and L, the color-singlet fermions and scalars of this model transform as follows.

$$
\begin{aligned}
(\nu_i, l_i)_L &\sim (\underline{3}, 1)\,, \\
l_{1R} &\sim (\underline{1}, 1), \\
l_{2R} &\sim (\underline{1}', 1), \\
l_{3R} &\sim (\underline{1}'', 1), \\
N_{iR} &\sim (\underline{3}, 0), \\
\Phi_i = (\phi_i^+, \phi_i^0) &\sim (\underline{3}, 0)\,, \\
\eta = (\eta^+, \eta^0) &\sim (\underline{1}, -1)\,.
\end{aligned}
$$

Hence its Lagrangian has the invariant terms

$$
\begin{aligned}
\mathcal{L} =& \frac{1}{2} M N_{iR}^2 + f \overline{N}_{iR} \left(\nu_{iL} \eta^0 - l_{iL} \eta^+ \right) \\
& + h_{ijk} \overline{(\nu_i, l_i)}_L l_{jR} \Phi_k + \mathrm{H.\,c.},
\end{aligned}
$$

where

$$
h_{i1k} = h_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},
$$

$$
h_{i2k} = h_2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix},
$$

$$
h_{i3k} = h_3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}.
$$

3. Model of nearly degenerate neutrino masses

The important assumptions to let you help understand the Lagrangian.

1. I limited the energy scale for N_{iR} to be about TeV.
2. I have four Higgs doublets$(\Phi_1, \Phi_2, \Phi_3, \eta)$.
3. I assigned the lepton number for N_{iR} to be 0 instead of 1.
4. I assigned the lepton number for η to be -1 instead of 0.

$$
\begin{aligned}
(\nu_i, l_i)_L &\sim (\underline{3}, 1)\,, \\
l_{1R} &\sim (\underline{1}, 1), \\
l_{2R} &\sim (\underline{1}', 1), \\
l_{3R} &\sim (\underline{1}'', 1), \\
N_{iR} &\sim (\underline{3}, 0), \\
\Phi_i = \left(\phi_i^+, \phi_i^0\right) &\sim (\underline{3}, 0)\,, \\
\eta = \left(\eta^+, \eta^0\right) &\sim (\underline{1}, -1)\,.
\end{aligned}
$$

3, 4 are related to a way writing down a new type of each Yukawa term.

3. Model of nearly degenerate neutrino masses

In order to know what this Lagrangian tells us, let's look into the Lagrangian in detail.

- Majorana masses for heavy r-h neutrinos

$$\mathcal{L} = \frac{1}{2} M N_{iR}^2 + f \overline{N}_{iR} \left(\nu_{iL} \eta^0 - l_{iL} \eta^+ \right) + h_{ijk} \overline{(\nu_i, l_i)}_L l_{jR} \Phi_k + \mathrm{H.\,c.},$$

- Dirac masses for l-h neutrinos
- Dirac masses for l-h charged leptons

First of all, let me contemplate for the third term in the above Lagrangian.

$$\mathcal{L} = \cdots + \cdots + h_{ijk} \overline{(\nu_i, l_i)}_L l_{jR} \Phi_k + \mathrm{H.\,c.},$$

3. Model of nearly degenerate neutrino masses

Let me contemplate for the charged-lepton terms in the Lagrangian.

$$\mathcal{L} = \cdots + \cdots + h_{ijk}\overline{(\nu_i, l_i)}_L l_{jR}\Phi_k + \mathrm{H.c.},$$

Expanding the third term, it becomes as below($\langle\phi_i^0\rangle = v_i$).

$$\begin{aligned}\mathcal{L}_{\text{third}} =& h_{ijk}\overline{(\nu_i, l_i)}_L l_{jR}\Phi_k \\ =& h_1 v_1 \bar{l}_{1L} l_{1R} + h_2 v_1 \bar{l}_{1L} l_{2R} + h_3 v_1 \bar{l}_{1L} l_{3R} \\ &+ h_1 v_2 \bar{l}_{2L} l_{1R} + h_2 v_2 \omega \bar{l}_{2L} l_{2R} + h_3 v_2 \omega^2 \bar{l}_{2L} l_{3R} \\ &+ h_1 v_3 \bar{l}_{3L} l_{1R} + h_2 v_3 \omega^2 \bar{l}_{3L} l_{2R} + h_3 v_3 \omega \bar{l}_{3L} l_{3R} \\ &+ \mathrm{H.c.},\end{aligned}$$

Let me see the above expanded form linking $\bar{l}_{iL}$ to l_{jR} $(i,\, j = 1, 2, 3)$ to the matrix form.

$$\mathcal{M}_l = \begin{pmatrix} h_1 v_1 & h_2 v_1 & h_3 v_1 \\ h_1 v_2 & h_2 v_2 \omega & h_3 v_2 \omega^2 \\ h_1 v_3 & h_2 v_3 \omega^2 & h_3 v_3 \omega \end{pmatrix}$$

3. Model of nearly degenerate neutrino masses

Let me see the above expanded form linking $\bar{l}_{iL}$ to l_{jR} $(i,\, j = 1, 2, 3)$ to the matrix form.

$$\mathcal{M}_l = \begin{pmatrix} h_1 v_1 & h_2 v_1 & h_3 v_1 \\ h_1 v_2 & h_2 v_2 \omega & h_3 v_2 \omega^2 \\ h_1 v_3 & h_2 v_3 \omega^2 & h_3 v_3 \omega \end{pmatrix}$$

If $v_1 = v_2 = v_3 = v$, then $\mathcal{M}_l$ is easily diagonalized:

$$U_L^\dagger M_l U_R = \begin{pmatrix} \sqrt{3} h_1 v & 0 & 0 \\ 0 & \sqrt{3} h_2 v & 0 \\ 0 & 0 & \sqrt{3} h_3 v \end{pmatrix} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

where

$$U_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \qquad U_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. Model of nearly degenerate neutrino masses

The 6×6 Majorana mass matrix spanning $(\overline{\nu_e}, \overline{\nu_\mu}, \overline{\nu_\tau}, N_1, N_2, N_3)$ is then given by

$$\mathcal{M}_{(\overline{\nu},N)} = \begin{pmatrix} 0 & U_L^\dagger f u \\ U_L^* f u & M \end{pmatrix}$$

Hence the 3×3 see-saw mass matrix for $(\nu_e, \nu_\mu, \nu_\tau)$ becomes

$$\mathcal{M}_\nu = \frac{f^2 u^2}{M} U_L^T U_L = \frac{f^2 u^2}{M} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \boxed{0} & \boxed{1} \\ 0 & \boxed{1} & \boxed{0} \end{pmatrix} \Longrightarrow$$

ν_μ mixes maximally with ν_τ

This matrix shows that ν_μ mixes maximally with ν_τ, but since all physical neutrino masses are degenerate, there are no neutrino oscillations.

3. Model of nearly degenerate neutrino masses

This below matrix shows that ν_μ mixes maximally with ν_τ, but since all physical neutrino masses are degenerate, there are no neutrino oscillations.

$$\mathcal{M}_\nu = \frac{f^2 u^2}{M} U_L^T U_L = \frac{f^2 u^2}{M} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \Longrightarrow$$

ν_μ mixes maximally with ν_τ

To break the degeneracy, the term MN_{iR}^2 may be substituted for arbitrary soft terms of the form $M_{ij}N_{iR}N_{jR}$ in the Lagrangian.

$$\mathcal{L} = \frac{1}{2} M N_{iR}^2 + f \overline{N}_{iR} \left(\nu_{iL} \eta^0 - l_{iL} \eta^+ \right) + h_{ijk} \overline{(\nu_i, l_i)}_L l_{jR} \Phi_k + \mathrm{H.\,c.},$$

$$\downarrow$$

$$\mathcal{L} = \frac{1}{2} M_{ij} N_{iR} N_{jR} + f \overline{N}_{iR} \left(\nu_{iL} \eta^0 - l_{iL} \eta^+ \right) + h_{ijk} \overline{(\nu_i, l_i)}_L l_{jR} \Phi_k + \mathrm{H.\,c.},$$

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4. Phenomenological consequences

- The minimal standard model → one Higgs scalar doublet
- This suggested A_4 model → $\Phi_{1,2,3}$, and η

The interplay between Φ_i and η is the same as in Ref[1], which allows $u = \langle \eta^0 \rangle$ to be small.

The corresponding A_4-invariant Higgs potential containing Φ is given by

$$
\begin{aligned}
V =& m^2 \sum_i \Phi_i^\dagger \Phi_i + \frac{1}{2}\lambda_1 \left(\sum_i \Phi_i^\dagger \Phi_i \right)^2 \\
&+ \lambda_2 \left(\Phi_1^\dagger \Phi_1 + \omega^2 \Phi_2^\dagger \Phi_2 + \omega \Phi_3^\dagger \Phi_3 \right) \left(\Phi_1^\dagger \Phi_1 + \omega \Phi_2^\dagger \Phi_2 + \omega^2 \Phi_3^\dagger \Phi_3 \right) \\
&+ \lambda_3 \left[\left(\Phi_2^\dagger \Phi_3 \right) \left(\Phi_3^\dagger \Phi_2 \right) + \left(\Phi_3^\dagger \Phi_1 \right) \left(\Phi_1^\dagger \Phi_3 \right) + \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) \right] \\
&+ \left\{ \frac{1}{2}\lambda_4 \left[\left(\Phi_2^\dagger \Phi_3 \right)^2 + \left(\Phi_3^\dagger \Phi_1 \right)^2 + \left(\Phi_1^\dagger \Phi_2 \right)^2 \right] + \mathrm{H.\,c.} \right\}
\end{aligned}
$$

[1] E. Ma, Phys. Rev. Lett **86**, 2502 (2001)

4. Phenomenological consequences

Before to go on, let me compare the A_4-invariant Higgs potential to the suggested Higgs potential by reference 1.

The corresponding Higgs potential by reference 1 is given by(one Φ, one η)

$$\begin{aligned} V\left(\Phi,\eta\right) =& m_1^2\Phi^\dagger\Phi + m_2^2\eta^\dagger\eta + \frac{1}{2}\lambda_1\left(\Phi^\dagger\Phi\right)^2 + \frac{1}{2}\lambda_2\left(\eta^\dagger\eta\right)^2 \\ &+ \lambda_3\left(\Phi^\dagger\Phi\right)\left(\eta^\dagger\eta\right) + \lambda_4\left(\Phi^\dagger\eta\right)\left(\eta^\dagger\Phi\right) \\ &+ \mu_{12}^2\left(\Phi^\dagger\eta + \eta^\dagger\Phi\right) \end{aligned}$$

The corresponding A_4-invariant Higgs potential is given by(three Φ, one η)

$$\begin{aligned} V\left(\Phi_i,\eta\right) =& m_i^2\Phi_i^\dagger\Phi_i + m_4^2\eta^\dagger\eta + \frac{1}{2}\lambda_i\left(\Phi_i^\dagger\Phi_i\right)^2 + \frac{1}{2}\lambda_4\left(\eta^\dagger\eta\right)^2 \\ &+ \lambda_5\left(\Phi_i^\dagger\Phi_i\right)\left(\eta^\dagger\eta\right) + \lambda_6\left(\Phi_i^\dagger\eta\right)\left(\eta^\dagger\Phi_i\right) \\ &+ \mu_{i4}^2\left(\Phi_i^\dagger\eta + \eta^\dagger\Phi_i\right) \end{aligned}$$

4. Phenomenological consequences

Before to go on, let me compare the A_4-invariant Higgs potential to the suggested Higgs potential by reference 1.

The corresponding A_4-invariant Higgs potential is given by(three Φ, one η)

$$\begin{aligned} V(\Phi_i,\eta) =& m_i^2 \Phi_i^\dagger \Phi_i + m_4^2 \eta^\dagger \eta + \frac{1}{2}\lambda_i \left(\Phi_i^\dagger \Phi_i\right)^2 + \frac{1}{2}\lambda_4 \left(\eta^\dagger \eta\right)^2 \\ &+ \lambda_5 \left(\Phi_i^\dagger \Phi_i\right)\left(\eta^\dagger \eta\right) + \lambda_6 \left(\Phi_i^\dagger \eta\right)\left(\eta^\dagger \Phi_i\right) \\ &+ \mu_{i4}^2 \left(\Phi_i^\dagger \eta + \eta^\dagger \Phi_i\right) \end{aligned}$$

The corresponding A_4-invariant Higgs potential containing Φ is given by(three Φ)

$$\begin{aligned} V(\Phi_i) =& m^2 \sum_i \Phi_i^\dagger \Phi_i + \frac{1}{2}\lambda_1 \left(\sum_i \Phi_i^\dagger \Phi_i\right)^2 \\ &+ \lambda_2 \left(\Phi_1^\dagger \Phi_1 + \omega^2 \Phi_2^\dagger \Phi_2 + \omega \Phi_3^\dagger \Phi_3\right)\left(\Phi_1^\dagger \Phi_1 + \omega \Phi_2^\dagger \Phi_2 + \omega^2 \Phi_3^\dagger \Phi_3\right) \\ &+ \lambda_3 \left[\left(\Phi_2^\dagger \Phi_3\right)\left(\Phi_3^\dagger \Phi_2\right) + \left(\Phi_3^\dagger \Phi_1\right)\left(\Phi_1^\dagger \Phi_3\right) + \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right)\right] \\ &+ \left\{\frac{1}{2}\lambda_4 \left[\left(\Phi_2^\dagger \Phi_3\right)^2 + \left(\Phi_3^\dagger \Phi_1\right)^2 + \left(\Phi_1^\dagger \Phi_2\right)^2\right] + \mathrm{H.c.}\right\} \end{aligned}$$

4. Phenomenological consequences

Let $\langle \phi_i^0 \rangle = v_i$, then the minimum of $V(\Phi_i)$ is

$$\begin{aligned} V_{min}(\Phi_i) = & m^2 \left(|v_1|^2 + |v_2|^2 + |v_3|^2\right) + \frac{1}{2}\lambda_1 \left(|v_1|^2 + |v_2|^2 + |v_3|^2\right)^2 \\ & + \lambda_2 \left(|v_1|^2 + \omega^2|v_2|^2 + \omega|v_3|^2\right)\left(|v_1|^2 + \omega|v_2|^2 + \omega^2|v_3|^2\right) \\ & + \lambda_3 \left(|v_2|^2|v_3|^2 + |v_3|^2|v_1|^2 + |v_1|^2|v_2|^2\right) \\ & + \left\{\frac{1}{2}\lambda_4 \left[\left(v_2^*\right)^2 v_3^2 + \left(v_3^*\right)^2 v_1^2 + \left(v_1^*\right)^2 v_2^2\right] + \text{c. c.}\right\} \end{aligned}$$

The minimization conditions on v_i are given by

$$\begin{aligned} 0 = \frac{\partial V_{min}}{\partial v_1^*} = & m^2 v_1 + \lambda_1 v_1 \left(|v_1|^2 + |v_2|^2 + |v_3|^2\right) \\ & + \lambda_2 v_1 \left(2|v_1|^2 - |v_2|^2 - |v_3|^2\right) + \lambda_3 v_1 \left(|v_2|^2 + |v_3|^2\right) \\ & + \lambda_4 v_1^* \left(v_2^2 + v_3^2\right), \end{aligned}$$

and other similar equations. Hence the solution

$$v_1 = v_2 = v_3 = v = \sqrt{\frac{-m^2}{3\lambda_1 + 2\lambda_3 + 2\lambda_4}}$$

is allowed if λ_4 is real.

4. Phenomenological consequences

The mass-squared matrices in the $\mathrm{Re}\,\phi_i^0$, $\mathrm{Im}\,\phi_i^0$, and $\phi_i^\pm$ bases are all of the form

$$\mathcal{M}^2 = \begin{pmatrix} a & b & b \\ b & a & b \\ b & b & a \end{pmatrix} = \begin{pmatrix} a+2b & 0 & 0 \\ 0 & a-b & 0 \\ 0 & 0 & a-b \end{pmatrix}$$

where

- $\mathrm{Re}\,\phi_i^0 \ : \quad a = 2\left(\lambda_1 + 2\lambda_2\right) v^2, \quad b = 2\left(\lambda_1 - \lambda_2 + \lambda_3 + \lambda_4\right) v^2,$
- $\mathrm{Im}\,\phi_i^0 \ : \quad a = -4\lambda_4 v^2, \quad b = 2\lambda_4 v^2,$
- $\phi_i^\pm \ : \quad a = -2\left(\lambda_3 + \lambda_4\right) v^2, \quad b = \left(\lambda_3 + \lambda_4\right) v^2.$

The eigenvalues of $\mathcal{M}^2$ are $a+2b$, $a-b$, and $a-b$.

4. Phenomenological consequences

Hence $(\Phi_1 + \Phi_2 + \Phi_3)/\sqrt{3}$ has the properties of the standard-model Higgs doublet with mass-squared eigenvalues

- $2\left(3\lambda_1 + 2\lambda_3 + 2\lambda_4\right) v^2$ for $\mathrm{Re}\left(\phi_1^0 + \phi_2^0 + \phi_3^0\right)/\sqrt{3}$,
- 0 for $\mathrm{Im}\left(\phi_1^0 + \phi_2^0 + \phi_3^0\right)/\sqrt{3}$,
- 0 for $\left(\phi_1^\pm + \phi_2^\pm + \phi_3^\pm\right)/\sqrt{3}$.

The two other linear combinations are mass degenerate in each sector with mass-squared eigenvalues given by

- $M_R^2 = 2\left(3\lambda_2 - \lambda_3 - \lambda_4\right) v^2$,
- $M_I^2 = -6\lambda_4 v^2$,
- $M_\pm^2 = -3\left(\lambda_3 + \lambda_4\right) v^2$.

	$a+2b$	$a-b$	$a-b$
$\mathrm{Re}\,\phi_i^0$	$2\left(3\lambda_1 + 2\lambda_3 + 2\lambda_4\right) v^2$	$2\left(3\lambda_2 - \lambda_3 - \lambda_4\right) v^2$	$2\left(\lambda_2 - \lambda_3 - \lambda_4\right) v^2$
$\mathrm{Im}\,\phi_i^0$	0	$-6\lambda_4 v^2$	$-6\lambda_4 v^2$
$\phi_i^\pm$	0	$-3\left(\lambda_3 + \lambda_4\right) v^2$	$-3\left(\lambda_3 + \lambda_4\right) v^2$

4. Phenomenological consequences

The distinct phenomenological signatures of our A_4 model are thus given by the two new Higgs doublets. They are predicted to be pairwise degenerate in mass and their Yukawa interactions are given by

$$\mathcal{L}_{int} = \left(\frac{m_\tau}{v} \overline{(\nu_e, e)}_L \tau_R + \frac{m_\mu}{v} \overline{(\nu_\tau, \tau)}_L \mu_R + \frac{m_e}{v} \overline{(\nu_\mu, \mu)}_L e_R \right) \Phi' + \left(\frac{m_\tau}{v} \overline{(\nu_\mu, \mu)}_L \tau_R + \frac{m_\mu}{v} \overline{(\nu_e, e)}_L \mu_R + \frac{m_e}{v} \overline{(\nu_\tau, \tau)}_L e_R \right) \Phi'' + \mathrm{H.\,c.},$$

where

- $\Phi' = \frac{1}{\sqrt{3}} \left(\Phi_1 + \omega \Phi_2 + \omega^2 \Phi_3 \right),$
- $\Phi'' = \frac{1}{\sqrt{3}} \left(\Phi_1 + \omega^2 \Phi_2 + \omega \Phi_3 \right),$

4. Phenomenological consequences

This means that the lepton flavor is necessarily violated and serves as an unmistakable prediction of this model.

Using the below Lagrangian,

$$\mathcal{L}_{int} = \Big(\frac{m_\tau}{v}\overline{(\nu_e, e)}_L \tau_R + \frac{m_\mu}{v}\overline{(\nu_\tau, \tau)}_L \mu_R + \frac{m_e}{v}\overline{(\nu_\mu, \mu)}_L e_R\Big)\,\Phi' + \Big(\frac{m_\tau}{v}\overline{(\nu_\mu, \mu)}_L \tau_R + \frac{m_\mu}{v}\overline{(\nu_e, e)}_L \mu_R + \frac{m_e}{v}\overline{(\nu_\tau, \tau)}_L e_R\Big)\,\Phi'' + \mathrm{H.c.},$$

we find that the most prominent (with strength $m_\tau m_\mu / v^2$) exotic decays of this model are

$$\tau_R^- \to \mu_L^- \mu_R^- e_R^+, \quad \tau_R^- \to \mu_L^- \mu_L^+ e_L^-,$$

through $(\phi'')^0$ exchange.

- $|\tau_R^- \to \mu_L^- \mu_R^- e_R^+| \propto M_0^{-2} = M_R^{-2} + M_I^{-2}$
- $|\tau_R^- \to \mu_L^- \mu_L^+ e_L^-| \propto M_1^{-2} = |M_R^{-2} - M_I^{-2}|$

Hence,

$$B\left(\tau^{-} \to \mu^{-}\mu^{-}e^{+}\right) = \left(\frac{9m_{\tau}^{2}m_{\mu}^{2}}{M_{0}^{4}}\right)\left(\frac{v_{0}^{2}}{3v^{2}}\right)^{2} B\left(\tau \to \mu\nu\nu\right),$$

where $v_0 = \left(2\sqrt{2}G_F\right)^{-1/2}$ and $3v^2 < v_0^2$. Using $B\left(\tau \to \mu\nu\nu\right) = 0.174$, we find

$$B\left(\tau^{-} \to \mu^{-}\mu^{-}e^{+}\right) = 5.5 \times 10^{-10}\left(\frac{v_{0}^{2}}{3v^{2}}\right)^{2}\left(\frac{100\,\mathrm{GeV}}{M_{0}}\right)^{4}$$

as compared to the experimental upper bound of 1.5×10^{-6}.

Similarly, $B\left(\tau_{R}^{-} \to \mu_{L}^{-}\mu_{L}^{+}e_{L}^{-}\right)$ is also given as below

$$B\left(\tau^{-} \to \mu^{-}\mu^{+}e^{-}\right) = \left(\frac{9m_{\tau}^{2}m_{\mu}^{2}}{M_{1}^{4}}\right)\left(\frac{v_{0}^{2}}{3v^{2}}\right)^{2} B\left(\tau \to \mu\nu\nu\right),$$

as compared to the experimental upper bound of 1.8×10^{-6}.

4. Phenomenological consequences

From the Lagrangian, there are also tree-level contributions to τ and μ decays through charged-scalar exchange.

$$\mathcal{L}_{int} = \left(\frac{m_\tau}{v}\overline{(\nu_e, e)}_L \tau_R + \frac{m_\mu}{v}\overline{(\nu_\tau, \tau)}_L \mu_R + \frac{m_e}{v}\overline{(\nu_\mu, \mu)}_L e_R\right)\Phi'$$
$$+ \left(\frac{m_\tau}{v}\overline{(\nu_\mu, \mu)}_L \tau_R + \frac{m_\mu}{v}\overline{(\nu_e, e)}_L \mu_R + \frac{m_e}{v}\overline{(\nu_\tau, \tau)}_L e_R\right)\Phi'' + \mathrm{H.\,c.},$$

For example,

$$\mu_R^- \to e_R^- \nu_\tau \overline{\nu_\mu}, \quad \mu_R^- \to e_R^- \nu_e \overline{\nu_\tau},$$

through $(\phi')^\pm$ and $(\phi'')^\pm$ exchange, respectively.

However, these amplitudes are completely negligible since

- they are proportional to $m_\mu m_e$ and
- add incoherently to the dominant $\mu_L^- \to e_L^- \nu_\mu \overline{\nu_e}$

The same holds true for τ decays, but to a lesser extent.

4. Phenomenological consequences

Consider next the muon anomalous magnetic moment, which receives a contribution proportional to m_τ^2 from $(\phi'')^0$.

A straightforward calculation yields

$$\Delta a_\mu = \frac{G_F m_\tau^2}{4\sqrt{2}\pi^2}\left(\frac{m_\mu^2}{M_0^2}\right)\left(\frac{v_0^2}{3v^2}\right) = 7.4\times 10^{-13}\left(\frac{v_0^2}{3v^2}\right)\left(\frac{100\,\mathrm{GeV}}{M_0}\right)^2,$$

as compared to the possible discrepancy[2] of $(426 \pm 165)\times 10^{-11}$, based on the recent experimental measurement[3].

Hence, the contribution to Δa_μ from the $\mathcal{L}_{int}$ is negligible, and the latter's theoretical explanation remains that of η and N exchange as proposed in Ref.[4]

[2] A. Czarnecki and W.J. Marciano, Phys. Rev. D **64**, 013014 (2001)
[3] H.N. Brown *et al.*, Phys. Rev. Lett. **86**, 2227 (2001)
[4] E. Ma and M. Raidal, Phys. Rev. Lett. **87**, 011802 (2001)

4. Phenomenological consequences

Radiative lepton-flavor-changing decays (i.e., $\tau \to \mu\gamma$, $\tau \to e\gamma$, $\mu \to e\gamma$) through η and N exchange are suppressed by the near degeneracy of the neutrino mass matrix[5].

However, they also receive contributions from $\mathcal{L}_{int}$. The most prominent process is actually $\mu \to e\gamma$ from $(\phi')^0$ exchange, with an amplitude given by

$$\mathcal{A} = \frac{e}{32\pi^2}\frac{m_\tau^2}{M_{eff}^2}\frac{m_\mu}{v^2}\epsilon^\alpha q^\beta \bar{e}\sigma_{\alpha\beta}\left(\frac{1+\gamma_5}{2}\right)\mu,$$

where

$$\frac{1}{M_{eff}^2} = \frac{1}{M_R^2}\left(\ln\frac{M_R^2}{m_\tau^2} - \frac{3}{2}\right) - \frac{1}{M_I^2}\left(\ln\frac{M_I^2}{m_\tau^2} - \frac{3}{2}\right).$$

Hence

$$B(\mu \to e\gamma) = \frac{27\alpha}{8\pi}\frac{m_\tau^4}{M_{eff}^4}\left(\frac{v_0^2}{3v^2}\right)^2.$$

Using the experimental upper bound[6] of 1.2×10^{-11}, we find $M_{eff} > 284\,\mathrm{GeV}(v_0/\sqrt{3}v)$.

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[5] E. Ma and M. Raidal, Phys. Rev. Lett. **87**, 011802 (2001)

[6] M.L. Brooks *et al.*, Phys. Rev. Lett. **83**, 1521 (1999).

Table of Contents

5. Concluding remarks

1. I have shown how nearly degenerate neutrino masses can be obtained in the context of a softly and spontaneously broken discrete A_4 (tetrahedral) symmetry while allowing realistic charged-lepton masses.
2. In addition to the standard model particles, we have three heavy neutral right-handed singlet fermions N_i at the TeV scale or below, whose decay into charged leptons would map out the neutrino mass matrix.
3. The nearly mass-degenerate N_i can explain the possible discrepancy of the muon anomalous magnetic moment.
4. The three new Higgs scalar doublets Φ_i of this model have distinct experimental signatures. One combination, i.e., $(\Phi_1 + \Phi_2 + \Phi_3)/\sqrt{3}$ behaves like the standard model Higgs doublet, except that it couples only to leptons.
5. The other two, i.e., Φ' and Φ'', are predicted to be pairwise mass degenerate and have precisely determined flavor-changing couplings. They are consistent with all present experimental bounds and amenable to experimental discovery below a TeV.