An Introduction to Grand Unified Theories

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LECTURE – 2

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Contents

Introduction

Need to go beyond SM

- Standard Model (SM) is *not* the final theory of particle physics.

- SM has excellent experimental agreement, but many shortcomings also:
  - gravity not included,
  - no explanation for neutrino mass and neutrino oscillation,
  - absence of candidates for dark matter,
  - no logic for several generations of fermions and their mass hierarchy,
  - no physical understanding of the plethora of parameters in the Lagrangian,
  - absence of any mechanism to generate observed baryon asymmetry in our universe,
  - no explanation for the observed smallness of CP violating θ term QCD.

- We shall study grand unification and grand unified theories (GUTs) with an eye on how they help alleviate some of the above ailments of SM.

Introduction

What is grand unification?

- Does the SM gauge group $G_{\rm SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$ provide an unified description of electroweak phenomena?
- $SU(2)_L \times U(1)_Y$ is product of two *disconnected* sets of gauge transformations: $SU(2)_L$ comes with coupling strength g and $U(1)_Y$ with g' which are not related by theory, i.e. the ratio,

$$g'/g = \tan\theta_W,$$

  has to fixed experimentally.
- A truly unified theory of electroweak interaction would use only one independent coupling constant instead of the above two.
- The strong interaction is also not unified with electroweak interaction in SM.
- The primary aim of grand unification is to describe strong, weak and electromagnetic interactions by a unified coupling constant via embedding SM gauge group inside a larger symmetry group.
- **Grand Unification $\equiv$ Unification of strong and electroweak interactions (without gravity)**

Introduction

Some excellent resources to start with:

- Paul Langacker, *Grand Unified Theories and Proton Decay*, Phys. Rep. **72**, 185 (1981).
- G. Ross, *Grand Unified Theories* (Benjamin, 1985).
- C. Patrignani *et al.* (Particle Data Group), Chapter 16, Chin. Phys. C, **40**, 100001 (2016).

- **Basic idea:** The SM gauge group G_{SM} is embedded in a larger symmetry group G. So strong and electroweak interactions are merged into one interaction and are described by one coupling constant.

- **Basic goal:** To break the unifying group G into G_{SM} by spontaneous symmetry breaking(s).

- **Constraint:** The unifying group G must be large enough to contain G_{SM} as a subgroup. Some possible unifying groups are,
  - Infinite series of groups: $SU(n)$, $SO(n)$, $Sp(2n)$,
  - Exceptional groups: F_4, E_6, E_7, E_8.

- **Byproducts:**
  - Additional symmetries of G restrict some arbitrary features of SM, e.g. family structure and fermion mass hierarchy.
  - New gauge bosons mediating new symmetries emerge which predict some observable phenomena.

- **Assume:**
  - Total number of fermions in our theory $= N_f$.
    $\therefore$ total number of left-handed and right-handed fermions $= 2N_f$.
  - $\exists\, G \supset G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$.
    G has a single coupling constant g_G.
    So the other coupling constants g_S, g, and g' are related to g_G.
- **Aim:** To find out how the coupling constants are related to one another.[1]
- **Step 1:** $\exists$ a maximal unifying group $G_{max} = SU(2N_f) \supset G \supset G_{\text{SM}}$. Algebraic relationships amongst coupling constants derived in G_{max} is automatically valid in G.
- **Step 2:** Denote the $2N_f \times 2N_f$ dimensional diagonal generators of G_{max}, $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ by T_G, T_C, T_L and T_Y respectively. $T_G^{(\alpha)}$ denotes the αth generator of G. Similarly, $T_A^{(i)}$ with $A \in \{C, L, Y\}$ denotes the ith generator of $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$.

[1] For details see, Jatinder K. Bajaj and G. Rajasekaran, "The electroweak mixing angle in unified gauge theories", Pramana, Vol.14, No.5, May 1980, pp.395-409.

- **Step 3:** It is known that $g_G T_G^{(\alpha)} = g_C T_C^{(i)}$, $g_G T_G^{(\beta)} = g T_L^{(j)}$ and $g_G T_G^{(\gamma)} = g' T_Y^{(k)}$. There is no reason why α, β, and γ (or i, j and k) should be same.
- **Step 4:** Also $T_G^{(\alpha)}$ and $T_A^{(i)}$ are normalized such that $\mathrm{Tr}\left(T_G^{(\alpha)}\, T_G^{(\alpha)}\right)$ is independent of α and $\mathrm{Tr}\left(T_A^{(i)}\, T_A^{(i)}\right)$ is independent of i, so that:

$$\mathrm{Tr}\left(T_G^{(\alpha)}\, T_G^{(\beta)}\right) = \delta^{\alpha\beta}\,\mathrm{Tr}\,(T_G)^2\,, \qquad \mathrm{Tr}\left(T_A^{(i)}\, T_A^{(j)}\right) = \delta^{ij}\,\mathrm{Tr}\,(T_A)^2\,.$$

$$\Longrightarrow g_G^2\,\mathrm{Tr}\,(T_G)^2 = g_S^2\,\mathrm{Tr}\,(T_C)^2 = g^2\,\mathrm{Tr}\,(T_L)^2 = g'^2\,\mathrm{Tr}\left(T_Q\right)^2,$$

$$\Longrightarrow \frac{g_G^2}{g_A^2} = \frac{\mathrm{Tr}\,(T_A)^2}{\mathrm{Tr}\,(T_G)^2} = \frac{\alpha_G}{\alpha_A}, \text{ where } \alpha_X = \frac{g_X^2}{4\pi} \text{ for } X \in \{G, C, L, Y\}.$$

Here for $A = C, L, Y$ we have $g_A = g_S, g, g'$ and $\alpha_A = \alpha_S, \alpha_L, \alpha_Y$ respectively.

$$\therefore \frac{\alpha_Y}{\alpha_S} = \frac{\mathrm{Tr}\,(T_C)^2}{\mathrm{Tr}\,(T_Y)^2} \text{ and } \frac{\alpha_Y}{\alpha_L} = \frac{\mathrm{Tr}\,(T_L)^2}{\mathrm{Tr}\,(T_Y)^2} = \tan^2\theta_W.$$

Generic features of grand unification

Ratios of Coupling constants in the Unifying Limit

- **Step 5:** Since $U(1)_{em} \subset SU(2)_L \times U(1)_Y$ we can construct a $2N_f \times 2N_f$ dimensional diagonal generator T_Q of $U(1)_{em}$ out of electric charges of all the fermions. The fine structure constant for $U(1)_{em}$ is denoted by α and following the previous step we get,

$$\frac{\alpha}{\alpha_L} = \frac{\mathrm{Tr}\,(T_L)^2}{\mathrm{Tr}\left(T_Q\right)^2} = \sin^2\theta_W.$$

- **Predicting ratios of coupling constants and finding** $\sin\theta_W$**:** For this we must proceed as follows,
  - construct the required generators,
  - square the generators and evaluate their trace,
  - predict the ratio of coupling constants/fine structure constants from the relations derived before.

- **Note:** The ratios of coupling constants as predicted above are valid at the unification scale. To determine the ratios at other energy scales, renormalization effects must be considered.

Generic features of grand unification

Ratios of Coupling constants in the Unifying Limit: **Example with doublets of fermions**

- **Assumption:** Quarks and leptons form doublets under the unifying group, i.e. for quarks we have $\begin{pmatrix} u^\alpha \\ d^\alpha \end{pmatrix}$ and for leptons we have $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$. Our discussion is independent of number of generations.
- **Generators:** 16×16 diagonal matrices,

$$T_Q = \text{diag}\left(\underbrace{\overbrace{\frac{2}{3},\frac{2}{3},\frac{2}{3}}^{u_L},\overbrace{-\frac{1}{3},-\frac{1}{3},-\frac{1}{3}}^{d_L},\overbrace{0}^{\nu_{e_L}},\overbrace{-1}^{e_L^-}}_{L},\underbrace{\overbrace{\frac{2}{3},\frac{2}{3},\frac{2}{3}}^{u_R},\overbrace{-\frac{1}{3},-\frac{1}{3},-\frac{1}{3}}^{d_R},\overbrace{0}^{\nu_{e_R}},\overbrace{-1}^{e_R^-}}_{R}\right)$$

$$T_L = \text{diag}\left(\underbrace{\overbrace{\frac{1}{2},\frac{1}{2},\frac{1}{2}}^{u_L},\overbrace{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}}^{d_L},\overbrace{\frac{1}{2}}^{\nu_{e_L}},\overbrace{-\frac{1}{2}}^{e_L^-}}_{L},\underbrace{\overbrace{0,0,0}^{u_R},\overbrace{0,0,0}^{d_R},\overbrace{0}^{\nu_{e_R}},\overbrace{0}^{e_R^-}}_{R}\right)$$

$$T_C = \text{diag}\left(\underbrace{\overbrace{\frac{1}{2},-\frac{1}{2},0}^{u_L},\overbrace{\frac{1}{2},-\frac{1}{2},0}^{d_L},\overbrace{0}^{\nu_{e_L}},\overbrace{0}^{e_L^-}}_{L},\underbrace{\overbrace{\frac{1}{2},-\frac{1}{2},0}^{u_R},\overbrace{\frac{1}{2},-\frac{1}{2},0}^{d_R},\overbrace{0}^{\nu_{e_R}},\overbrace{0}^{e_R^-}}_{R}\right)$$

- **Traces:**

$$\mathrm{Tr}\left(T_Q\right)^2 = \frac{16}{3}, \quad \mathrm{Tr}\left(T_L\right)^2 = 2, \quad \mathrm{Tr}\left(T_C\right)^2 = 2.$$

- **Predictions:**

$$\frac{\alpha}{\alpha_S} = \frac{\mathrm{Tr}\left(T_C\right)^2}{\mathrm{Tr}\left(T_Q\right)^2} = \frac{3}{8},$$

$$\frac{\alpha}{\alpha_L} = \frac{\mathrm{Tr}\left(T_L\right)^2}{\mathrm{Tr}\left(T_Q\right)^2} = \sin^2\theta_W = \frac{3}{8} = 0.375.$$

- **Note 1:** The above results are independent of the actual unifying group as long as it is a subgroup of G_{max} which in this case is $SU(16)$ and as long as the doublet scheme is maintained.
- **Note 2:** Large renormalization corrections are required to bring down the value of $\sin^2\theta_W$ from 0.375 to the experimentally measured value 0.23126.

Generic features of grand unification

Ratios of Coupling constants in the Unifying Limit: **Example with triplets of fermions**

- **Assumption:** The quarks and leptons form triplets under G, i.e. for quarks we have $\begin{pmatrix} u^\alpha \\ d^\alpha \\ x^\alpha \end{pmatrix}$ and for leptons we have $\begin{pmatrix} \nu_e \\ e^- \\ E^+ \end{pmatrix}$.
- **Generators:** 24×24 diagonal matrices,

$$T_Q = \mathrm{diag}\Bigg(\underbrace{\overbrace{\tfrac{2}{3},\tfrac{2}{3},\tfrac{2}{3}}^{u_L},\overbrace{-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3}}^{d_L},\overbrace{-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3}}^{x_L},\overbrace{0}^{\nu_{e_L}},\overbrace{-1}^{e_L^-},\overbrace{+1}^{E_L^+}}_{L},\underbrace{\overbrace{\tfrac{2}{3},\tfrac{2}{3},\tfrac{2}{3}}^{u_R},\overbrace{-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3}}^{d_R},\overbrace{-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3}}^{x_R},\overbrace{0}^{\nu_{e_R}},\overbrace{-1}^{e_R^-},\overbrace{+1}^{E_R^+}}_{R}\Bigg)$$

$$T_L = \mathrm{diag}\Bigg(\underbrace{\overbrace{\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}}^{u_L},\overbrace{-\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2}}^{d_L},\overbrace{0,0,0}^{x_L},\overbrace{\tfrac{1}{2}}^{\nu_{e_L}},\overbrace{-\tfrac{1}{2}}^{e_L^-},\overbrace{0}^{E_L^+}}_{L},\underbrace{\overbrace{0,0,0}^{u_R},\overbrace{0,0,0}^{d_R},\overbrace{0,0,0}^{x_R},\overbrace{0}^{\nu_{e_R}},\overbrace{0}^{e_R^-},\overbrace{0}^{E_R^+}}_{R}\Bigg)$$

$$T_C = \mathrm{diag}\Bigg(\underbrace{\overbrace{\tfrac{1}{2},-\tfrac{1}{2},0}^{u_L},\overbrace{\tfrac{1}{2},-\tfrac{1}{2},0}^{d_L},\overbrace{\tfrac{1}{2},-\tfrac{1}{2},0}^{x_L},\overbrace{0}^{\nu_{e_L}},\overbrace{0}^{e_L^-},\overbrace{0}^{E_L^+}}_{L},\underbrace{\overbrace{\tfrac{1}{2},-\tfrac{1}{2},0}^{u_R},\overbrace{\tfrac{1}{2},-\tfrac{1}{2},0}^{d_R},\overbrace{\tfrac{1}{2},-\tfrac{1}{2},0}^{x_R},\overbrace{0}^{\nu_{e_R}},\overbrace{0}^{e_R^-},\overbrace{0}^{E_R^+}}_{R}\Bigg)$$

Generic features of grand unification

Ratios of Coupling constants in the Unifying Limit: **Example with triplets of fermions**

- **Traces:**

$$\mathrm{Tr}\left(T_Q\right)^2 = 8, \quad \mathrm{Tr}\left(T_L\right)^2 = 2, \quad \mathrm{Tr}\left(T_C\right)^2 = 3.$$

- **Predictions:**

$$\frac{\alpha}{\alpha_S} = \frac{\mathrm{Tr}\left(T_C\right)^2}{\mathrm{Tr}\left(T_Q\right)^2} = \frac{3}{8} = 0.375,$$

$$\frac{\alpha}{\alpha_L} = \frac{\mathrm{Tr}\left(T_L\right)^2}{\mathrm{Tr}\left(T_Q\right)^2} = \sin^2\theta_W = \frac{1}{4} = 0.25.$$

- **Note:** Here the renormalization corrections required are smaller in comparison with the doublet fermion scenario. *This example is for the purpose of illustration only.* We shall consider the doublet scenario in our discussion ahead.

Generic features of grand unification

Renormalization effects

- Once the unified symmetry is broken, the coupling constants undergo different renormalizations and hence their relevant ratios (at low energies) after symmetry breaking are different. Here we use renormalization group.
- **Assumption:** Unification group G breaks down to G_{SM} by a single step SSB at the energy scale M_U.
- For $q^2 = M_U^2$, we have
$$\frac{1}{\alpha_G\left(M_U^2\right)} = \frac{1}{\alpha_S\left(M_U^2\right)} = \frac{1}{\alpha_L\left(M_U^2\right)} = \frac{1}{\alpha_Y\left(M_U^2\right)} = \frac{3}{5}.$$
- **Renormalization group equations:** For $q^2 < M_U^2$,

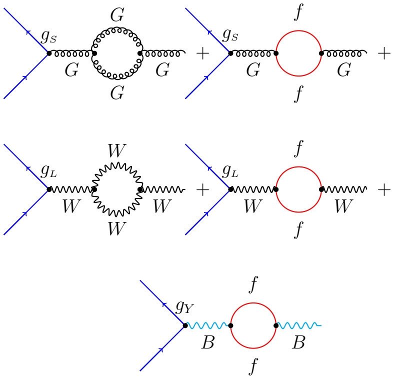
$$\frac{1}{\alpha_S\left(q^2\right)} = \frac{1}{\alpha_S\left(M_U^2\right)} - \beta_S \ln\frac{q^2}{M_U^2},$$
$$\frac{1}{\alpha_L\left(q^2\right)} = \frac{1}{\alpha_L\left(M_U^2\right)} - \beta_L \ln\frac{q^2}{M_U^2},$$
$$\frac{3}{5}\frac{1}{\alpha_Y\left(q^2\right)} = \frac{3}{5}\frac{1}{\alpha_Y\left(M_U^2\right)} - \frac{3}{5}\beta_Y \ln\frac{q^2}{M_U^2}.$$

- **β-functions:**

$$\beta_S = -\frac{11}{16\pi^2} + F, \quad \beta_L = -\frac{11}{16\pi^2}\frac{2}{3} + F, \quad \frac{3}{5}\beta_Y = 0 + F,$$

where F denotes the fermion contribution.

- Above results are obtained from the one-loop contributions from the following kind of diagrams:



❖ **Notes:**

❶ *Decoupling theorem* is invoked to ignore the contribution of heavy particles. New gauge bosons are all assumed to be sufficiently heavy so as not to contribute to these one-loop equations for $q^2 \ll M_U^2$.

❷ The non-abelian gauge-boson contributions lead to the negative numbers given for β_S and β_L, whereas there is no such contribution for the abelian case β_Y.

❸ The fermion contribution F is the same for all the three graphs, assuming that the complete fermion multiplet is light. We ignore Higgs contributions.

❹ We use the doublet scheme of fermions, in which case $\dfrac{\mathrm{Tr}\,(T_G)^2}{\mathrm{Tr}\,(T_Y)^2} = \dfrac{3}{5}$.

❺ Also we have assumed the usual normalization for the non-abelian generators: $\mathrm{Tr}\,(T_G)^2 = \mathrm{Tr}\,(T_C)^2 = \mathrm{Tr}\,(T_L)^2$.

Generic features of grand unification

Renormalization effects

- After algebraic simplification results for the ratios of the coupling constants in the low-energy region $M_L \approx 100$ GeV has the form:

$$\frac{\alpha}{\alpha_S} = \frac{3}{8} - \frac{33}{8\pi}\alpha \ln \frac{M_U}{M_L},$$

$$\sin^2\theta_W = \frac{3}{8} - \frac{55}{24\pi}\alpha \ln \frac{M_U}{M_L}.$$

- By rewriting the 1st expression above we get

$$\frac{3}{8}\frac{1}{\alpha} - \frac{1}{\alpha_S} = \frac{33}{8\pi} \ln \frac{M_U}{M_L}.$$

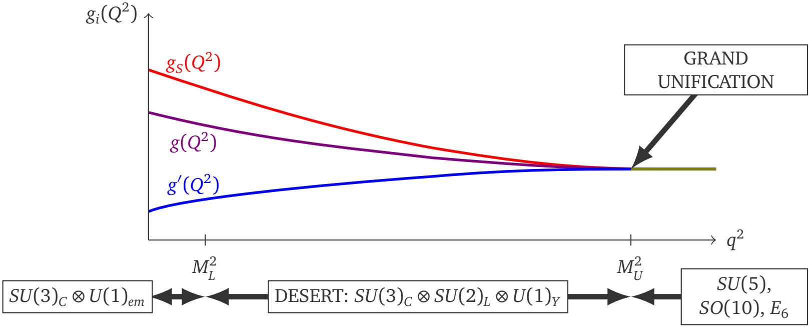
- By substituting the empirical values of the coupling constants for QCD and QED at the 'low' energy scale M_L:

$$\frac{1}{\alpha_S} \approx 5; \ \frac{3}{8}\frac{1}{\alpha} \approx 50; \ \frac{3}{8}\frac{1}{\alpha} - \frac{1}{\alpha_S} \approx 45.$$

  This leads to the unification scale:

$$M_U \approx 100 \text{ GeV} \times \exp\left(\frac{45 \times 8\pi}{33}\right) \approx 10^{15} \text{ GeV}.$$

- Using the predicted value of $M_U \approx 10^{15}$ GeV we get $\sin^2\theta_W \approx 0.21$.
- **Unification of coupling constants:**



Generic features of grand unification

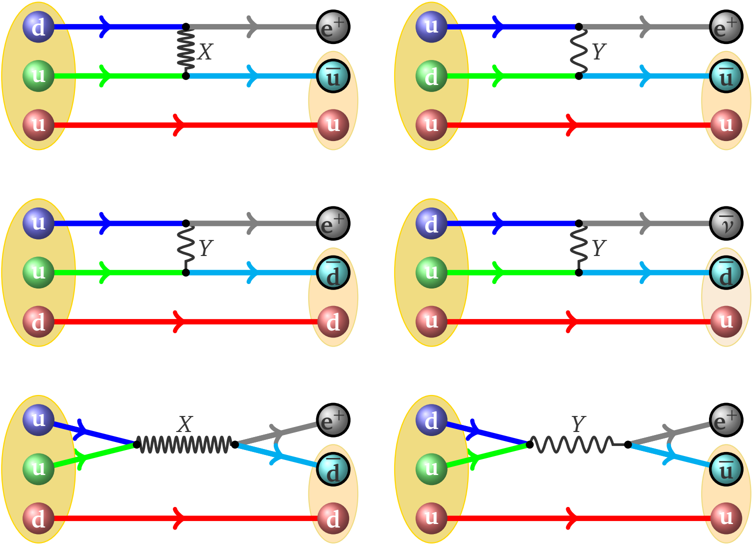
Phenomenological features: extra gauge bosons and proton decay

- During grand unification, we put quarks and leptons into the same representation of the unifying group G.
- This implies that there are gauge bosons (usually denoted by X, Y) that transform quarks to leptons and vice-versa (Baryon number is violated in such processes).
- These gauge bosons are called as **lepto-quarks** (if they mediate the transitions between quarks and leptons) or **diquarks** (if they mediate quark to quark transitions). Due to their presence, proton decay[2] is allowed in grand unified theories: $p \to \pi^0 e^+$.
- **Observation of proton decay would be a tell-tale signature of grand unification, just as observation of $0\nu\beta\beta$ would be a definitive signature of Majorana neutrinos.**

[2] See the paper: Paul Langacker (1981), "Grand Unified Theories And Proton Decay", Physics reports (Review Section of Physics letters) **72**, No.4, Pages 185-385, for more details and extensive discussions on proton decay.

Generic features of grand unification

Phenomenological features: extra gauge bosons and proton decay



Generic features of grand unification

Phenomenological features: extra gauge bosons and proton decay

- Let $M_X =$ mass of the lepto-quarks (or diquarks).
  $\therefore$ Amp$\left(p \to \pi^0 e^+\right) \sim g_G^2/M_X^2$, where g_G the coupling strength of the grand unifying gauge group.
- Neglecting the pion and positron masses in comparison to proton mass,

$$\Gamma_P = (g_G^2/M_X^2)^2 \times \left(\begin{array}{c} \text{Phase Space Factor} \\ \text{controlled by proton} \\ \text{mass } m_P \end{array} \right).$$

- By dimensional analysis, $\Gamma_P \sim \left(g_G^2/M_X^2\right)^2 m_P^5$.
- Putting $M_X \sim M_U \sim 10^{15}$ GeV, the mean life of proton comes out as

$$\tau_P \sim \frac{1}{\Gamma_P} \sim \frac{M_U^4}{m_p^5} \sim 10^{31} \text{ years} \tag{1}$$

  This is only an order of magnitude estimate. So far no proton decay has been seen experimentally.
- *Grand unification not only predicts the correct value of the electroweak mixing angle θ_W, but also the longevity of the proton.*

- The Georgi-Glashow (GG) $SU(5)$ model was the first attempt to embed the standard model in an underlying simple group[3]. The model is the simplest grand unified theory that is phenomenologically viable.
- **Gauge bosons** represented by the adjoint representation **24** and their decomposition under $SU(3)_C \times SU(2)_L$ are as follows:

$$\mathbf{24} = \underbrace{(8,1)}_{G^\alpha_\beta} + \underbrace{(1,3)}_{(W^\pm, W^0)} + \underbrace{(1,1)}_{B} + \underbrace{(3,2)}_{(X_\alpha, Y_\alpha)} + \underbrace{(3^*,2)}_{(\bar{X}^\alpha, \bar{Y}^\alpha)}$$

  where the entries (n_3, n_2) represent the representations under the $SU(3)_C$ and $SU(2)_L$ subgroups.
- The **24** contains an octet of gluons G^α_β, 4 electro-weak bosons ($W^\pm$, W^0, B) and 12 new bosons (X_α, Y_α, $\bar{X}^\alpha$, $\bar{Y}^\alpha$, where α is the color index) that carry both flavor and color, and hence they mediate Baryon number violating interactions.

[3] See the paper: G. Georgi, S.L. Glashow (1974). "Unity of All Elementary Particle Forces". Physical Review Letters **32**: 438-441.

Examples of grand unified theories

Example 1: Georgi-Glashow $SU(5)$ model

❖ **Fermions** of one family belong to $\mathbf{5}^* + \mathbf{10}$ and they are given below:

$$\mathbf{5}^* \equiv \begin{pmatrix} \nu_e & \\ & d^c_\alpha \\ e^- & \end{pmatrix}_L = \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \\ e^- \\ -\nu_e \end{pmatrix}_L$$

and

$$\mathbf{10} \equiv \begin{pmatrix} & u^\alpha & \\ e^+ & & u^c_\alpha \\ & d^\alpha & \end{pmatrix}_L = \begin{pmatrix} 0 & u^c_3 & -u^c_2 & -u^1 & -d^1 \\ -u^c_3 & 0 & u^c_1 & -u^2 & -d^2 \\ u^c_2 & -u^c_1 & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{pmatrix}_L$$

where the superscript c denotes charge-conjugate. Note that quarks, leptons and anti-quarks (u^c and d^c) appear together in the above representations.

- **Regarding spontaneous symmetry breaking:**
  - $SU(5)$ is spontaneously broken down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ by an adjoint (**24**) representation $\Phi_a^b(\Phi_a^a = 0)$ of Higgs fields (where $a, b = 1, 2, \cdots, 5$).
  - $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant components of Φ have very large VEVs $\approx \mathcal{O}\left(10^{14}\text{ GeV}\right)$.
  - The Φ therefore generates $M_X = M_Y \gtrsim 10^{14}$GeV, $M_W = M_Z = m_l = m_q = 0$. For $M_W^2 \ll q^2 \lesssim M_U^2$, $SU(3)_C \times SU(2)_L \times U(1)_Y$ is an approximately unbroken symmetry, and g_S, g, and g' evolve independently.
    $\because M_W^2 < q^2 \lesssim M_U^2$ is barren of any qualitatively new physics, it is called as the DESERT or PLATEAU.
  - A **5**-dimensional Higgs representation H^a with a much smaller value of VEV of the order of 100 GeV breaks the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry down further to $SU(3)_C \times U(1)_{em}$, generating $|M_X^2 - M_Y^2|^{1/2} \sim M_W \sim M_Z \sim \mathcal{O}(100\text{ GeV})$ and $m_l \neq 0$, $m_q \neq 0$.

- **Strong features:**
  - $SU(5)$ model has the minimal fermion and Higgs representations.
  - Quarks and lepton universality is inbuilt $\because$ the $\mathbf{5}^*$ and $\mathbf{10}$ representations of fermions contain both in the same family.
  - There is no room for a ν_R in the $\mathbf{5}^* + \mathbf{10}$ representations. Hence neutrinos remain massless, unless we include ν_R as an $SU(5)$ singlet.
  - Electric charge, originally a (continuous variable) generator of the $U(1)$ group, is a generator of $SU(5)$, and the commutation relations of this symmetry allow only discrete, rather than continuous, eigenvalues of electric charge. So charge quantization, in this model, occurs as a result of grand unification. The quark and lepton charges are related. The sum of charges of the particles in each multiplet is zero.
  - It predicts approximately correct value of $\sin^2\theta_W$.

- **Weak features:**
  - Each family is placed in a reducible $\mathbf{5}^* + \mathbf{10}$ representation. In larger groups each family is usually in an irreducible representation.
  - There is no explanation of why there are several families or of how many there are.
  - The model still has many free parameters. (The model with only one **24** and one **5** Higgs representation has: 1 gauge coupling, 1 θ parameter, 9 Higgs parameters (7 if $\Phi \to -\Phi$), 6 quark masses (from which the lepton masses are deduced), 6 mixing angles and 1 CP violating phase.)
  - The model predicts the existence of super heavy ($m \approx 10^{16}$ GeV) 't Hooft-Polyakov magnetic monopoles.
  - The model includes a desert between the W and X(or Y) masses.
- Minimal $SU(5)$ GUT is already ruled out by experimental data from proton decay experiments. However, some modified versions such as supersymmetric $SU(5)$ or flipped $SU(5)$ are still allowed by experimental data.

Examples of grand unified theories

Example 2: $SO(10)$ model

- Fritzsch and Minkowski proposed a grand unified theory[4] based on rank-5 group $SO(10)$.
- Each family of left-handed fermions is assigned to a 16 dimensional complex spinor σ_+. Consider two distinct subgroups of $SO(1))$, viz. $SU(5)$ and $SU(4)\times SU(2)\times SU(2)$. Under the $SU(5)$ subgroup the fermion spinor σ_+ for the first family decomposes as:

$$\mathbf{16} = \underbrace{\mathbf{5}^*}_{\begin{pmatrix} \nu_e & & \\ & & d^c_\alpha \\ e^- & & \end{pmatrix}_L} + \underbrace{\mathbf{10}}_{\begin{pmatrix} & u^\alpha & \\ e^+ & & u^c_\alpha \\ & d^\alpha & \end{pmatrix}_L} + \underbrace{\mathbf{1}}_{\nu^c_{eL}}.$$

- **Note:** σ_+ contains an $SU(5)$ singlet, an antineutrino $\nu^c_{eL} = C\bar{\nu}^T_R$. Hence, the $SO(10)$ will in general involve massive neutrinos.

[4]See the paper: Harald Fritzsch and Peter Minkowski (1975),"Unified Interactions of leptons and hadrons", Annals of Physics, Volume-93, Pages 193-266.

- Of the 45 generators of $SO(10)$, 24 are those of the $SU(5)$ subgroup. The rest 21 generators connect or distinguish between the $\mathbf{5}^*$, $\mathbf{10}$ and $\mathbf{1}$. In order to study their properties it is convenient to consider the subgroup decomposition:

$$\begin{aligned} SO(10) \supset & SO(6) \times SO(4) \\ & \sim SU(4)_C \times SU(2) \times SU(2) \\ & \supset SU(3)_C \times U(1)' \times SU(2)_L \times SU(2)_R, \end{aligned} \tag{2}$$

  where $SU(4)_C \supset SU(3)_C \times U(1)'$ can be considered an extended color group with leptons as fourth color.

- With respect to $SU(3)_C \times SU(2)_L \times SU(2)_R$, σ_+ decomposes as:

$$\mathbf{16} = \underbrace{(3,2,1)}_{\begin{pmatrix} u^\alpha \\ d^\alpha \end{pmatrix}_L} + \underbrace{(1,2,1)}_{\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L} + \underbrace{(3^*,1,2)}_{\begin{pmatrix} d^c_\alpha \\ -u^c_\alpha \end{pmatrix}_L} + \underbrace{(1,1,2)}_{\begin{pmatrix} e^+ \\ -\nu^c_e \end{pmatrix}_L}.$$

Examples of grand unified theories

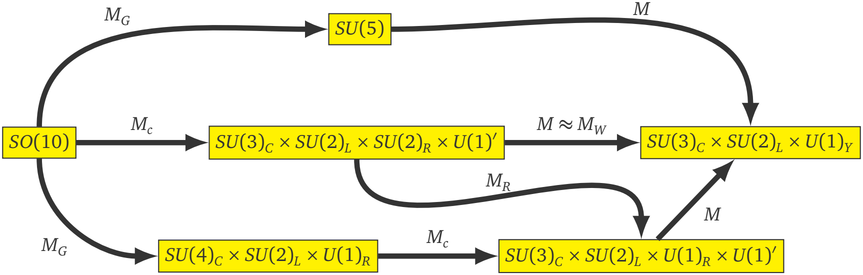
Example 2: $SO(10)$ model

❖ $SO(10)$ contains the left-right symmetric $SU(2)_L \times SU(2)_R \times U(1)'$ electro-weak group as a subgroup. In particular, the right-handed fermions (left-handed anti-fermions) transform nontrivially under $SU(2)_R$. The transformation properties of 45 gauge bosons under $SU(3)_C \times SU(2)_L \times SU(2)_R$ are:

$$\mathbf{45} = \underbrace{(8,1,1)}_{G^\alpha_\beta} + \underbrace{(1,3,1)}_{(W_L^{\pm,0})} + \underbrace{(1,1,3)}_{(W_R^{\pm,0})} + \underbrace{(1,1,1)}_{B}$$
$$+ \underbrace{(3^*,2,2)}_{\begin{pmatrix} X & \bar{Y}' \\ Y & \bar{X}' \end{pmatrix}} + \underbrace{(3,2,2)}_{\begin{pmatrix} X' & \bar{Y} \\ Y' & \bar{X} \end{pmatrix}} + \underbrace{(3,1,1)}_{X_S} + \underbrace{(3^*,1,1)}_{\bar{X}_S}.$$

❖ The fifteen bosons G^α_β, $W^{\pm,0}_{L,R}$ and B' are associated with $SU(3)_{color} \times SU(2)_L \times SU(2)_R \times U(1)'$. Now, there are weak bosons corresponding to both $SU(2)_L$ and $SU(2)_R$ and the lepto-quarks/diquarks X, Y are also larger in number.

Patterns of symmetry breaking in $SO(10)$



Examples of grand unified theories

Example 2: $SO(10)$ model

- The $SO(10)$ model is an attractive extension of the $SU(5)$ model in the sense that quarks and leptons are treated much more symmetrically.
- There is more freedom in choosing symmetry breaking patterns. So its difficult to uniquely predict $\sin^2\theta_W$ and proton life-time in $SO(10)$.
- $SO(10)$ is also left-right symmetric, while $SU(5)$ is not.
- $SO(10)$ allows for neutrino mass.
- No desert in between M_W and M_U.
- Some variants of $SO(10)$, for example supersymmetric $SO(10)$, are still allowed by the proton decay data.

Phenomenological influence of GUT inspired ideas

- The following two prominent ideas are currently resurrecting in view of some observed tensions in B decays:
  - Lepto-quarks,
  - W' and Z' bosons.
- The new models incorporating lepto-quarks and/or W', Z' bosons are, nevertheless, phenomenologically motivated and not necessarily involve unification of strong and electroweak coupling constants.
- We shall discuss only a few very recent applications of these ideas.

- Lepton universality check in $b \to s\ell^+\ell^-$ decays via R_K and R_{K^*}:

$$R_{K^{(*)}} = \frac{\mathrm{Br}\left(B \to K^{(*)}\mu^+\mu^-\right)}{\mathrm{Br}\left(B \to K^{(*)}e^+e^-\right)}$$

$$R_K^{\mathrm{SM}} = 1, \quad R_K^{\mathrm{exp}} = 0.745 \pm 0.09 \pm 0.036,$$

[R. Aaij *et al.* [LHCb] PRL 113, 151601 (2017)]

$$R_{K^*}^{\mathrm{SM}} = 1, \quad R_{K^*}^{\mathrm{exp}} = \begin{cases} 0.660^{+0.110}_{-0.070} \pm 0.024 & 4m_\mu^2 < q^2 < 1.1\mathrm{GeV}^2 \\ 0.685^{+0.113}_{-0.069} \pm 0.047 & 1.1\mathrm{GeV}^2 < q^2 < 6\mathrm{GeV}^2 \end{cases}$$

[S. Bifani, CERN seminar, April 18 (2017)]

- Both R_K and R_{K^*} differ from SM expectation by $\sim 2.4\sigma$ standard deviation.

Phenomenological influence of GUT inspired ideas

B decay anomalies

- Lepton universality check in $b \to c\ell^- \nu$ decays via R_D and R_{D^*}:

$$R_{D^{(*)}} = \frac{\mathrm{Br}\left(\bar{B} \to D^{(*)}\tau^- \nu\right)}{\mathrm{Br}\left(B \to D^{(*)}\ell^- \nu\right)}$$

$$R_D^{\mathrm{SM}} = 0.300 \pm 0.008, \quad R_D^{\mathrm{exp}} = 0.407 \pm 0.039 \pm 0.024,$$

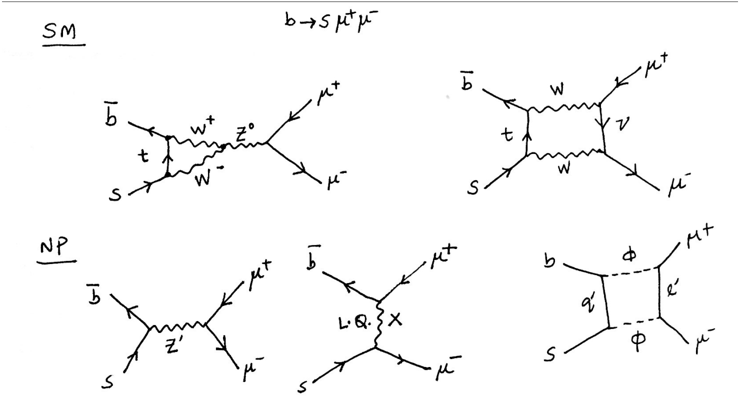
$$R_{D^*}^{\mathrm{SM}} = 0.252 \pm 0.003, \quad R_{D^*}^{\mathrm{exp}} = 0.304 \pm 0.013 \pm 0.007$$

[results of the heavy flavor averaging group (HFLAG)]

- The R_D and R_{D^*} differ from SM expectation by $\sim 2.0\sigma$ and $\sim 2.7\sigma$ standard deviations respectively. The combined significant of disagreement is 3.4σ.

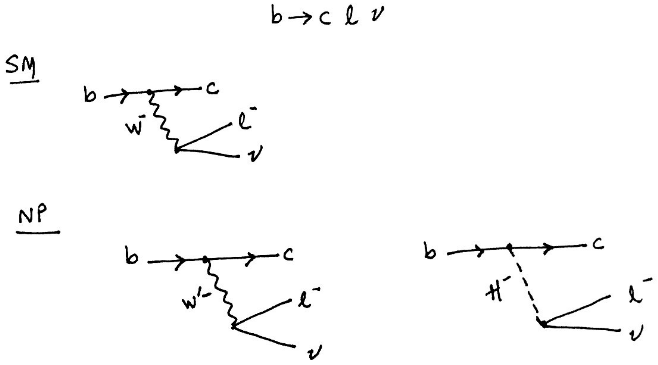
Phenomenological influence of GUT inspired ideas

SM and NP contributions in $b \to s\ell^+\ell^-$ processes



Phenomenological influence of GUT inspired ideas

SM and NP contributions in $b \to c\ell^- \nu$



Conclusion

- The idea of grand unification leads to very rich theory and phenomenology. All GUTs predict proton decay due to the presence of extra gauge bosons called lepto-quarks and/or diquarks.
- Grand unification has the ability to alleviate many ailments of the SM, but experimental data for proton decay measurements rule out the very simple GUT, the $SU(5)$ GUT.
- The concept of lepto-quarks, W' and Z' bosons which originally appeared in the literature in the context of GUTs are now being pursued for various other purposes unrelated to grand unification.

Thank you