Model-independent signatures of New Physics in $B \to D\ell^+\ell^-$ decays

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Plan of the talk

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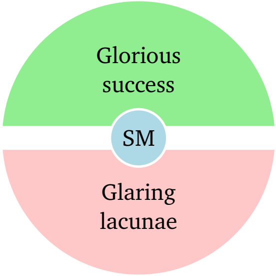

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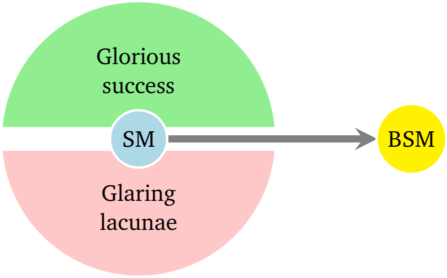

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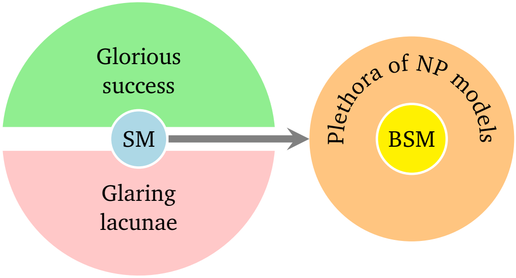

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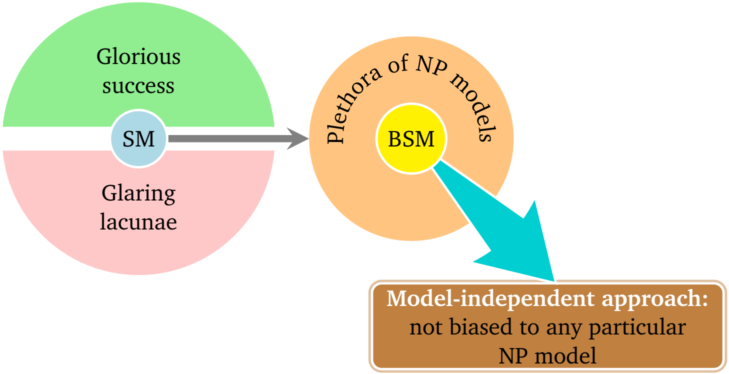

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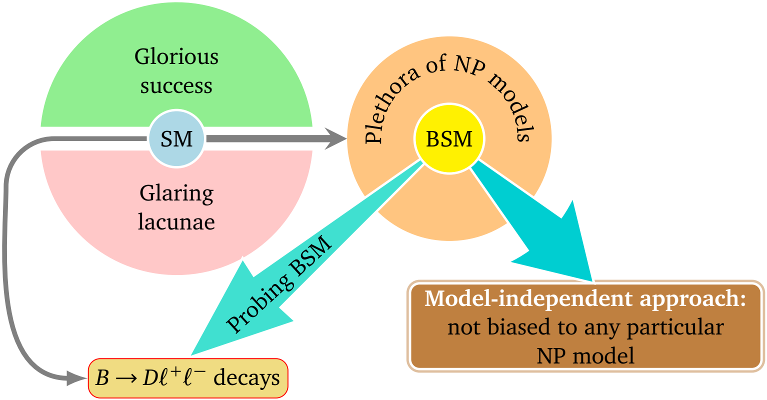

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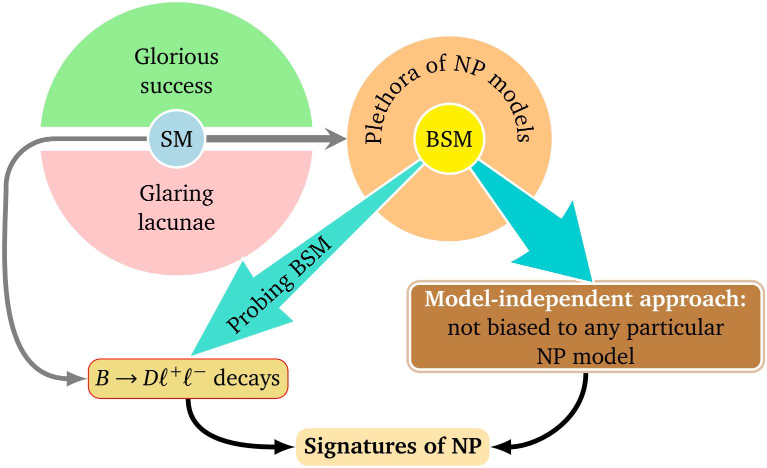

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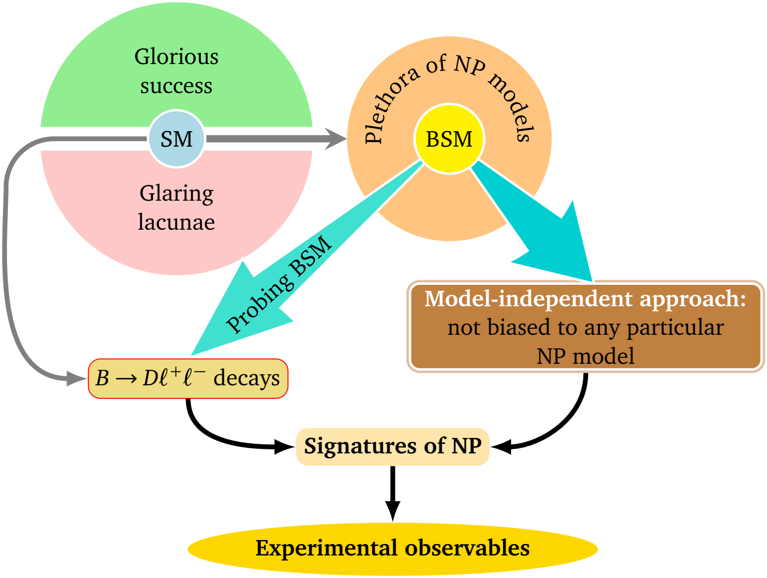
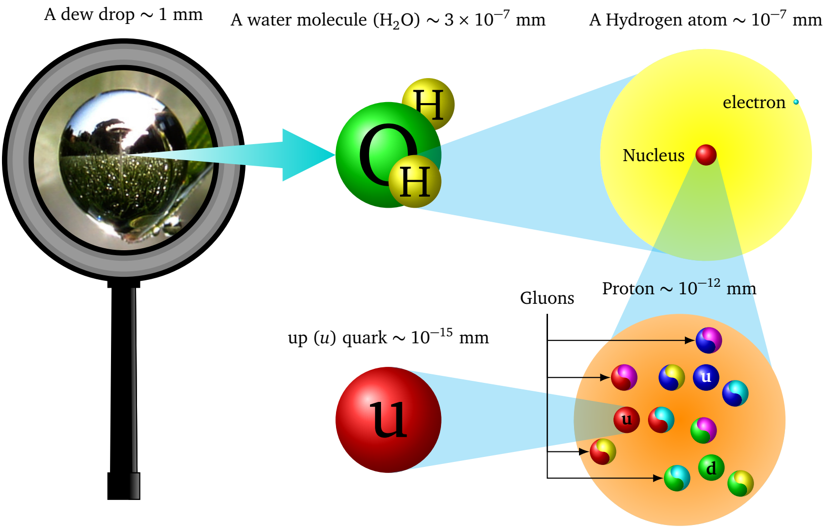


Elementary particle physics concerns with the most elementary constituents of matter and how they interact.



The standard model is currently the best description of most of our observations about elementary particles.

- "*Standard Model* (SM) of particle physics" is a misnomer – *it is not the final theory of particle physics* but it certainly is the most experimentally vindicated theory of elementary particles.

- SM has
  - phenomenologically motivated symmetries, fields, and Lagrangian,
  - well-defined calculational rules,
  - nice experimental agreement, as well as
  - many unexplained features.

SM encompasses all known elementary particles: quarks, leptons, photon, gluons, weak gauge bosons & Higgs.

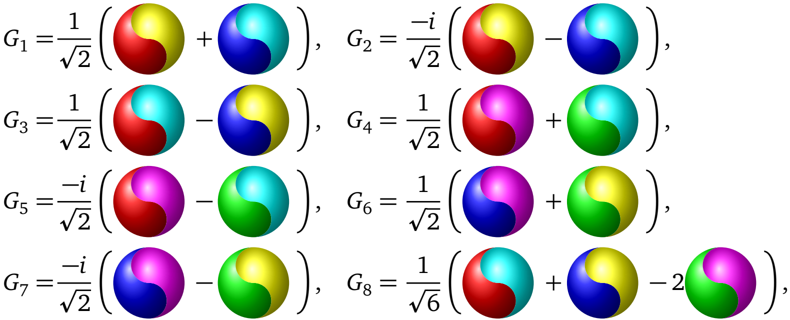
Fermions:

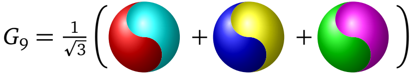
- Quarks: $\underbrace{u,\bar{u},d,\bar{d}}_{\text{Family I}},\underbrace{c,\bar{c},s,\bar{s}}_{\text{Family II}},\underbrace{b,\bar{b},t,\bar{t}}_{\text{Family III}}$. fractionally charged $(\pm\frac{2}{3},\pm\frac{1}{3})$.
  Strong and electroweak interactions. Color quantum number $\left(R \;\; \bar{R} \;\; G \;\; \bar{G} \;\; B \;\; \bar{B} \right)$. color confinement. asymptotic freedom. Baryon number $\pm\frac{1}{3}$. Strangeness $(s \to -1)$, Charmness $(c \to +1)$, Bottomness $(b \to -1)$.
- Leptons: $\underbrace{e^+,e^-,\nu_e,\bar{\nu}_e}_{\text{Family I}},\underbrace{\mu^+,\mu^-,\nu_\mu,\bar{\nu}_\mu}_{\text{Family II}},\underbrace{\tau^+,\tau^-,\nu_\tau,\bar{\nu}_\tau}_{\text{Family III}}$. Electroweak interactions. Lepton number. Neutrino oscillation.

SM encompasses all known elementary particles: quarks, leptons, photon, gluons, weak gauge bosons & Higgs.

Bosons:

- Gauge bosons: one massless photon (γ), eight massless gluons (G_i^μ, $i = 1, \ldots, 8$)





(This one is not allowed being color singlet), and three massive weak gauge bosons (W^+, W^-, Z^0).

- Higgs boson (H)

SM encompasses all known elementary particles: quarks, leptons, photon, gluons, weak gauge bosons & Higgs.

- The SM describes (i) Strong interaction (quarks and gluons), (ii) Electromagnetic interaction (all electrically charged particles and photon), (iii) Weak interaction (quarks, leptons and weak gauge bosons) and (iv) Higgs interactions.
- Foundation of SM is the symmetry group: $SU(3)_c \times SU(2)_L \times U(1)_Y$.
- $SU(3)_c \implies$ Quantum Chromodynamics (QCD) $\rightarrow$ strong interaction amongst quarks and gluons.
- $SU(2)_L \times U(1)_Y \implies$ Quantum Flavordynamics (QFD) $\rightarrow$ broken at a low energy scale ($\sim$ 100 GeV) via spontaneous symmetry breaking (SSB) involving the Higgs scalar $\rightarrow$ weak and electromagnetic interactions amongst quarks and leptons.
- Interaction of Higgs with itself, quarks, leptons and gauge bosons is also included in SM.

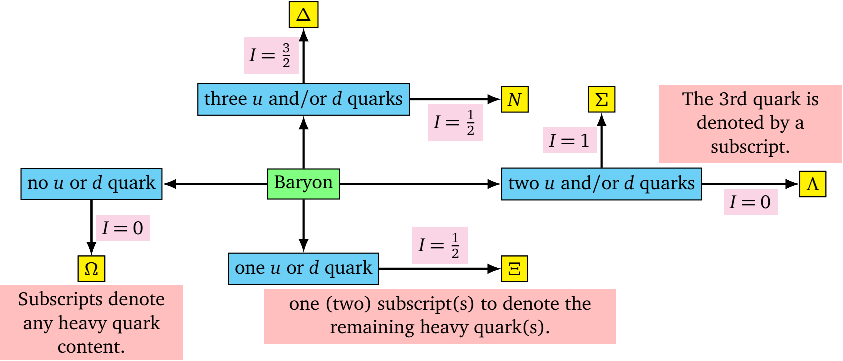
SM also explains very well many observations related to composite particles.

- SM explains properties (decay rates, production cross-sections, oscillations, CP violation) of a plethora of composite particles.
- Composite particles in SM are **hadrons** which are color singlets. Hadrons are made up of quark antiquark combinations.
  - **Mesons:** $q_i\overline{q_j}$. E.g. π, ρ, ω, ϕ, K, B, D, J/ψ, Υ etc.
  - **Baryons:** $q_iq_jq_k$ or $\overline{q_i}\ \overline{q_j}\ \overline{q_k}$. E.g. p, n, Σ, Δ, Ξ, Ω, Λ etc.
  - **Exotics:** tetraquark or pentaquark states.

SM also explains very well many observations related to composite particles.

❖ Nomenclature scheme:

- **Mesons:** Main symbol is an upper-case italic letter indicating the heavier quark (or $\overline{\text{quark}}$) as follows: $s \to \bar{K}$, $\bar{s} \to K$, $c \to D$, $\bar{c} \to \bar{D}$, $b \to \bar{B}$, $\bar{b} \to B$. Lighter quark identity is specified by a subscript.
- **Baryons:**



The interaction of SM fermions depends on whether they are left-handed or right-handed.

- Fermions (quarks and leptons) come in 3 families (*observation*).
- Taking helicity (L ≡ Left, R ≡ Right) components of fermions under $SU(2)_L$, we have,

Family	**Leptons**	**Quarks**
1st family	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, e^-_R, e^+_L, e^+_R, \overline{\nu}_{eR}$	$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix}_L, u_R, d_R, \overline{u}_R, \overline{d}_R$
2nd family	$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \mu^-_R, \mu^+_L, \mu^+_R, \overline{\nu}_{\mu R}$	$\begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} \bar{s} \\ \bar{c} \end{pmatrix}_L, c_R, s_R, \bar{c}_R, \bar{s}_R$
3rd family	$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \tau^-_R, \tau^+_L, \tau^+_R, \overline{\nu}_{\tau R}$	$\begin{pmatrix} t \\ b \end{pmatrix}_L, \begin{pmatrix} \bar{b} \\ \bar{t} \end{pmatrix}_L, t_R, b_R, \bar{t}_R, \bar{b}_R$

- Total number of fermions is $90 = 18$ leptons $+ (24 \times 3)$ quarks (considering 3 color possibilities for each quark).
- All neutrinos are left-handed and all anti-neutrinos are right handed.

SM Lagrangian: the most general, renormalizable & $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant Lagrangian

$$
\begin{aligned}
\mathscr{L}_{\rm SM} = & \underbrace{-\frac{1}{4}G^i_{\mu\nu}G^{i\,\mu\nu}}_{(I)} \underbrace{-\frac{1}{4}W^a_{\mu\nu}W^{a\,\mu\nu}}_{(II)} \underbrace{-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}}_{(III)} + \underbrace{\Theta\, G^i_{\mu\nu}\, G^i_{\alpha\beta}\, \epsilon^{\mu\nu\alpha\beta}}_{\text{The }\Theta\text{ term}}, \\
& + \underbrace{\sum_{n=1}^{3} \bar{q}^{(n)}_L \gamma^\mu \left(i\partial_\mu - g_3 \frac{\lambda^i}{2} G^i_\mu - g_2 \frac{\tau^a}{2} W^a_\mu - \frac{g_1}{6} B_\mu \right) q^{(n)}_L}_{(IV)} \\
& + \underbrace{\sum_{n=1}^{3} \bar{u}^{(n)}_R \gamma^\mu \left(i\partial_\mu - g_3 \frac{\lambda^i}{2} G^i_\mu - \frac{2g_1}{3} B_\mu \right) u^{(n)}_R}_{(V)} + \underbrace{\sum_{n=1}^{3} \bar{d}^{(n)}_R \gamma^\mu \left(i\partial_\mu - g_3 \frac{\lambda^i}{2} G^i_\mu + \frac{g_1}{3} B_\mu \right) d^{(n)}_R}_{(VI)}, \\
& + \underbrace{\sum_{n} \bar{l}^{(n)}_L \gamma^\mu \left(i\partial_\mu - g_2 \frac{\tau^a}{2} W^a_\mu + \frac{g_1}{2} B_\mu \right) l^{(n)}_L}_{(VII)} + \underbrace{\sum_{n} \bar{e}^{(n)}_R \gamma^\mu \left(i\partial_\mu + g_1 B_\mu \right) e^{(n)}_R}_{(VIII)} \\
& + \underbrace{\left| \left(i\partial_\mu - g_2 \frac{\tau^a}{2} W^a_\mu - g_1 \frac{1}{2} B_\mu \right) \Phi \right|^2}_{(IX)} + \underbrace{\mu^2 \Phi^\dagger \Phi - \frac{\lambda}{2} \left(\Phi^\dagger \Phi \right)^2}_{(X)} \\
& + \underbrace{\sum_{m,n} \left(\Gamma^u_{mn}\, \bar{q}^{(m)}_L\, \Phi^c\, u^{(n)}_R + \Gamma^d_{mn}\, \bar{q}^{(m)}_L\, \Phi\, d^{(n)}_R + \Gamma^e_{mn}\, \bar{l}^{(m)}_L\, \Phi\, e^{(n)}_R + \text{h.c.} \right)}_{(XI)}
\end{aligned}
$$

SM Lagrangian: the most general, renormalizable & $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant Lagrangian

$$G^i_{\mu\nu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu + g_3 f^{ijk} G^j_\mu G^k_\nu,$$
$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu,$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$
$$q^{(n)}_L = \left\{ q^{(1)}_L = \begin{pmatrix} u_\alpha \\ d_\alpha \end{pmatrix}_L, q^{(2)}_L = \begin{pmatrix} c_\alpha \\ s_\alpha \end{pmatrix}_L, q^{(3)}_L = \begin{pmatrix} t_\alpha \\ b_\alpha \end{pmatrix}_L \text{, with } \alpha = r, g, b \right\},$$
$$l^{(n)}_L = \left\{ l^{(1)}_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, l^{(2)}_L = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, l^{(3)}_L = \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \right\},$$
$$u^{(n)}_R = \left\{ u^{(1)}_R = u_{\alpha R}, u^{(2)}_R = c_{\alpha R}, u^{(3)}_R = t_{\alpha R} \text{, with } \alpha = r, g, b \right\},$$
$$d^{(n)}_R = \left\{ d^{(1)}_R = d_{\alpha R}, d^{(2)}_R = s_{\alpha R}, d^{(3)}_R = b_{\alpha R} \text{, with } \alpha = r, g, b \right\},$$
$$e^{(n)}_R = \left\{ e^{(1)}_R = e_R, e^{(2)}_R = \mu_R, e^{(3)}_R = \tau_R \right\}$$
$$\Phi = \begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \Longrightarrow \Phi^c = i\,\tau_2\,\Phi^*.$$

SM Lagrangian: the most general, renormalizable & $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant Lagrangian

1. g_1, g_2, and g_3 are the $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$ coupling constants.

2. $\lambda^i \equiv$ Eight Gell-Mann Matrices, and $\tau^a \equiv$ Three Pauli Matrices.

3. $f^{ijk} \equiv$ completely antisymmetric $\left(f^{ijk} = -f^{ikj} = f^{kij}\right)$ structure constants, $\left[\lambda^i, \lambda^j\right] = 2if^{ijk}\,\lambda^k$. The non-zero structure constants are, $f^{123} = 1, f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2}, f^{458} = f^{678} = \frac{\sqrt{3}}{2}$.

4. $\epsilon^{abc} \equiv$ Levi-Civita symbol, $\left[\tau^a, \tau^b\right] = 2i\epsilon^{abc}\,\tau^c$.

SM Lagrangian: the most general, renormalizable & $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant Lagrangian

1. Terms I, II, and III $\Longrightarrow$ kinetic energies + self-interactions of the pure gauge field part of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ non-abelian gauge theory.

2. Θ term violates P, T, and CP symmetries (for $\Theta \neq 0$). It contributes to the electric dipole moment (EDM) of neutron, d_n. There is stringent experimental limit on the possible value of EDM for neutron. The best experimental upper limit[1] amounts to $|d_n| < 2.9 \times 10^{-26} e$-cm. This gives an extremely small upper bound[2] on Θ: $|\Theta| < 0.7 \times 10^{-11}$.

[1] With 90% confidence level, Baker et.al., Phys. Rev. Lett. 97. (2006)
[2] Jihn E. Kim and Gianpaolo Carosi, Rev. Mod. Phys. **82**, 557-601 (2010).

SM Lagrangian: the most general, renormalizable & $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant Lagrangian

3. Terms *IV*, *V*, *VI*, *VII* and *VIII* $\Longrightarrow$ kinetic energies + interaction of fermions with the gauge fields.
4. No mass terms for the fermions[3]. Mass terms are generated via Brout-Englert-Higgs Mechanism.
5. Terms *IX*, *X* and *XI* $\Longrightarrow$ interactions of the Higgs field with itself, with the gauge bosons, and fermions $\Longrightarrow$ SSB, generation of masses of $W^{\pm}$, Z^0, quarks and leptons[4]. In the Higgs part of the Lagrangian (terms *IX* and *X*), we have $\mu^2 > 0$ and $\lambda > 0$ for SSB.
6. Coefficients Γ^x_{mn}, where $x \in \{u, d, e\}$, in term *XI* are Yukawa Couplings. These are arbitrary parameters in the SM. They couple the Right and Left helicities. Each of the Γ^x matrix is a 3×3 matrix of Yukawa Couplings, with the Γ^x_{mn} term giving the coupling between the generation m and the generation n.

[3] Such terms violate the $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariance of the Lagrangian $\mathscr{L}_{SM}$.

[4] Neutrinos are massless in SM. If we can put a term like $\Gamma^{\nu}_{mn}\, \bar{l}_{mL}\, \Phi\, \nu_{nR}$ in term *XI* of $\mathscr{L}_{SM}$, then neutrinos can get mass by SSB (**Dirac Mass**). Also all the gluons and the photon remain massless. However, mass of the Higgs boson can not be predicted in SM. It is sensitive to the shape of the scalar potential.

The Standard Model of particle physics is one of the most successful theories of our times.

The various interactions of elementary particles as observed in many particle physics experiments is very well described by the SM.

Some of the most cherished success stories of SM include,

- Prediction and discovery of the Higgs boson, $W^{\pm}$, Z^0, gluons, top, bottom and charm quarks.
- Quark mixing matrix and CP violation in meson sector.
- Prediction of one electroweak mixing angle.
- Precisely accouting for various decays of Z^0.
- Precise prediction of anomalous magnetic dipole moment of electron.

SM accomplishes an incomplete description of our observable universe at its most fundamental level.

The Standard Model of particle physics (SM) fails to answer,

1. What is the quantum description of gravity?
2. What are the constituents of dark matter, and what is dark energy?
3. How to explain the matter anti-matter asymmetry observed in our universe?
4. How do neutrinos get such tiny masses?
5. How to describe the observed muon anomalous magnetic dipole moment $(g-2)$?
6. Why is there no CP violation in strong interaction?
7. Is there any physical understanding of the plethora of SM parameters?
8. Why do quarks and leptons appear in three families?
9. How to unify the strong and electro-weak interactions?

… and so on.

Despite its glaring lacunae, SM is our best description of the experimentally observed zoo of elementary particles except the neutrinos.

With insufficient experimental guidance we are lost in the rain-forest of Beyond SM scenarios (New Physics).

We have many beyond standard model scenarios (New Physics possibilities):

- Grand Unified Theories ($SU(5)$, $SU(8)$, $SO(10)$, ...),
- Supersymmetry (MSSM, NMSSM, ...),
- Extra dimensions (Large ED, warped ED, universal ED, ...),
- Neutrino mass models (see-saw, inverse see-saw, ...),
- Dark matter models (WIMP, SIMP, Axions, ...),
- Two-Higgs-doublet model,
- Technicolor,
- Preonic models (Rishon model, Quantum Haplodynamics, ...),
- Quantum gravity (loop quantum gravity, SUGRA, ...),
- String theory, ... etc.

Experiment is the touchstone of all new physics possibilities.

Our best strategy to search for new physics is to look for its model-independent signatures.

With so many new physics (NP) models around, how do we figure out which model is the correct one.

Besides, how can we ensure that all conceivable NP possibilities have already been taken care of by existing NP models.

We need results from more particle physics experiments, concerning various processes that have not been considered so far, to get a better indication of the nature of NP.

Would it not be better to know how NP, without considering any specific NP model, would affect some processes in a very general manner (except enhancing the probability of their occurance)?

This is precisely the place where a model-independent analysis of NP plays a very significant role.

Heavy flavor physics is a very popular area where new physics contributions are favourable.

The most popular flavor physics probe for NP has been the $b \to s\ell^+\ell^-$ transition, e.g. in $B \to K^{(*)}\ell^+\ell^-$ decays. Such decays are primarily interesting because they involve FCNC and large contributions from electroweak loop (Penguin and box) diagrams which are fertile grounds for many NP models.

However, in analysis of $B \to K^{(*)}\ell^+\ell^-$ decays, the weak annihilation and weak exchange diagrams are neglected in comparison with the penguin diagrams as they are suppressed by $\mathcal{O}\left(\Lambda_{QCD}/m_B\right)$.

What would happen if some NP effect enhances the weak annihilation or weak exchange diagrams? How can we possibly search for the nature of any such NP?

$B \to D\ell^+\ell^-$ decays provide a fine avenue to look for signatures of new physics in a very model-independent manner.

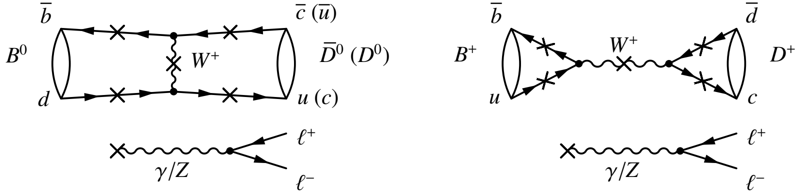
Processes such as $B^0 \to \bar{D}^0\ell^+\ell^-$ which predominantly take place via weak exchange diagrams, can have large branching ratio $\mathcal{O}\left(10^{-5}\right)$ due to large Wilson coefficients[†].

[†]C. S. Kim, R. H. Li and Y. Li, JHEP **1110**, 152 (2011).

So such decays can be looked at experimentally (e.g. at LHCb and Belle-II), to find out the nature of any NP that can possibly contribute here.

We are interested in finding out the signature of any NP contributing to $B \to D\ell^+\ell^-$ decays in a model-independent manner.

The interaction Hamiltonian for $B \to D\ell^+\ell^-$ involves six-fermion interaction.



There are three currents involved in these decays:

- two charged currents involving the valence quarks,
- one neutral current involving the charged leptons.

Six-fermion interactions (mass dimension 9) are usually suppressed than four-fermion interactions (mass dimension 6).

The $B^0 \to \overline{D}^0 \ell^+ \ell^-$ decay mode is less suppressed in comparison with $B^0 \to D^0 \ell^+ \ell^-$ and $B^+ \to D^+ \ell^+ \ell^-$.

- In $B^0 \to \bar{D}^0 \ell^+ \ell^-$ the two quark transitions are $\bar{b} \to \bar{c} W^+$ (which is CKM-suppressed) and $d \to u W^-$ (which is CKM-favoured).
- However, in $B^+ \to D^+ \ell^+ \ell^-$ and $B^0 \to D^0 \ell^+ \ell^-$ the two quark transitions are $\bar{b} \to \bar{u} W^+$ and $d \to c W^-$, both of which are CKM-suppressed.

So, it is expected that $B^0 \to \bar{D}^0 \ell^+ \ell^-$ would have larger branching ratio than $B^+ \to D^+ \ell^+ \ell^-$ and $B^0 \to D^0 \ell^+ \ell^-$. Therefore, in our discussion ahead we shall concentrate on $B^0 \to \bar{D}^0 \ell^+ \ell^-$ decay alone.

Nevertheless, all our arguments and conclusions are valid for other $B \to D \ell^+ \ell^-$ decay modes as well.

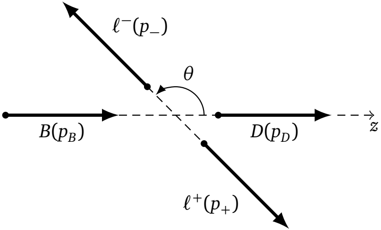
The most general model-independent amplitude for $B \to D\ell^+\ell^-$ decays has a few general form factors.

$$
\begin{aligned}
\mathcal{M}\left(B \to D\ell^+\ell^-\right) = \frac{G_F^2}{m_B}\Big\{ & F_S\left(\bar{\ell}\ \mathbf{1}\ \ell\right) && \text{Scalar} \\
& + F_P\left(\bar{\ell}\ \gamma^5\ \ell\right) && \text{Pseudoscalar} \\
& + \left(F_V^+ p_\alpha + F_V^- q_\alpha\right)\left(\bar{\ell}\ \gamma^\alpha\ \ell\right) && \text{Vector} \\
& + \left(F_A^+ p_\alpha + F_A^- q_\alpha\right)\left(\bar{\ell}\ \gamma^\alpha\ \gamma^5\ \ell\right) && \text{Axialvector} \\
& + F_{T_1}\ p_\alpha\ q_\beta\left(\bar{\ell}\ \sigma^{\alpha\beta}\ \ell\right) && \text{MDM} \\
& + F_{T_2}\ p_\alpha\ q_\beta\left(\bar{\ell}\ \sigma^{\alpha\beta}\ \gamma^5\ \ell\right)\Big\}. && \text{EDM}
\end{aligned}
$$

Here F denotes form factors, $p \equiv p_B + p_D$ and $q \equiv p_B - p_D = p_+ + p_-$.
All NP information is contained in the form factors.

We analyse the $B \to D\ell^+\ell^-$ decays in the Gottfried-Jackson frame.

Gottfried-Jackson frame



Notation á la Mandelstam:

$$s = (p_+ + p_-)^2 = (p_B - p_D)^2,$$
$$t = (p_D + p_-)^2 \equiv a - b\cos\theta,$$
$$u = (p_D + p_+)^2 \equiv a + b\cos\theta.$$

where

$$a = \frac{1}{2}\left(m_B^2 + m_D^2 + 2m_\ell^2 - s\right),$$
$$b = \frac{1}{2}\left(\sqrt{\lambda\left(m_B^2, m_D^2, s\right)\left(1 - 4m_\ell^2/s\right)}\right),$$

with the Källén function $\lambda(x, y, z)$ defined as

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx).$$

The angular distribution for $B \to D\ell^+\ell^-$ decays in the Gottfried-Jackson frame has three distinct parts.

$$\frac{d^2\Gamma}{ds\, d\cos\theta} = \frac{b\sqrt{s}}{128\ \pi^3\ m_B^2\left(m_B^2 - m_D^2 + s\right)}\left(T_0 + T_1\ \cos\theta + T_2\ \cos^2\theta\right).$$

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$$\begin{aligned}
T_0 = {} & 2s\left(|F_S|^2 + |F_P|^2\right) + 2\left(\left|F_V^+\right|^2 + \left|F_A^+\right|^2\right)\lambda\left(m_B^2, m_D^2, s\right) \\
& - 8m_\ell\Big(\left(m_B^2 - m_D^2\right)\mathrm{Re}\left(F_P F_A^{+*}\right) + \mathrm{Re}\left(F_P F_A^{-*}\right)s \\
& \qquad - \mathrm{Im}\left(F_V^+ F_{T_1}^*\right)\lambda\left(m_B^2, m_D^2, s\right)\Big) \\
& + 8m_\ell^2\Big(-|F_S|^2 + \left(\left|F_A^-\right|^2 - \left|F_A^+\right|^2\right)s \\
& \qquad + 2\left(m_B^2 + m_D^2\right)\left|F_A^+\right|^2 + \left|F_{T_1}\right|^2\lambda\left(m_B^2, m_D^2, s\right) \\
& \qquad + 2\left(m_B^2 - m_D^2\right)\mathrm{Re}\left(F_A^+ F_A^{-*}\right)\Big),
\end{aligned}$$

The angular distribution for $B \to D\ell^+\ell^-$ decays in the Gottfried-Jackson frame has three distinct parts.

$$\frac{d^2\Gamma}{ds\, d\cos\theta} = \frac{b\sqrt{s}}{128\ \pi^3\ m_B^2\left(m_B^2 - m_D^2 + s\right)}\left(T_0 + T_1\ \cos\theta + T_2\ \cos^2\theta\right).$$

$$\begin{aligned}
T_1 = 8b\Big(s\left(\mathrm{Im}\left(F_S F_{T_1}^*\right) + \mathrm{Im}\left(F_P F_{T_2}^*\right)\right) \\
- 2m_\ell\Big(\mathrm{Im}\left(F_A^+ F_{T_2}^*\right)\left(m_B^2 - m_D^2\right) \\
- \mathrm{Re}\left(F_S F_V^{+*}\right) + \mathrm{Im}\left(F_A^- F_{T_2}^*\right)s\Big)\Big),
\end{aligned}$$

$$T_2 = -8b^2\left(\left|F_V^+\right|^2 + \left|F_A^+\right|^2 - s\left(\left|F_{T_1}\right|^2 + \left|F_{T_2}\right|^2\right)\right),$$

Considering leptons massless, immensely simplifies the angular distribution.

In the massless lepton limit (i.e. $m_\ell \to 0$), which is a very good approximation at B mass scale, we have

$$
\begin{aligned}
T_0\Big|_{m_\ell=0} &= 2s\left(|F_S|^2 + |F_P|^2\right) + 2\left(\left|F_V^+\right|^2 + \left|F_A^+\right|^2\right)\lambda\left(m_B^2, m_D^2, s\right), \\
T_1\Big|_{m_\ell=0} &= 4s\Big(\mathrm{Im}\left(F_{T_1}F_S^*\right) + \mathrm{Im}\left(F_{T_2}F_P^*\right)\Big)\sqrt{\lambda\left(m_B^2, m_D^2, s\right)}, \\
T_2\Big|_{m_\ell=0} &= -2\left(\left|F_V^+\right|^2 + \left|F_A^+\right|^2\right)\lambda\left(m_B^2, m_D^2, s\right) \\
&\quad + 2s\left(\left|F_{T_1}\right|^2 + \left|F_{T_2}\right|^2\right)\lambda\left(m_B^2, m_D^2, s\right),
\end{aligned}
$$

In SM γ and Z boson give rise to final $\ell^+\ell^-$. So, in SM we have $F_S = F_P = F_{T_1} = F_{T_2} = 0$. This implies,

$$
T_0 = -T_2 = 8b^2\left(\left|F_A^+\right|^2 + \left|F_V^+\right|^2\right), \qquad \text{and} \qquad T_1 = 0.
$$

The three distinct parts of the angular distribution of $B \to D\ell^+\ell^-$ decays satisfy two important equations.

- $\boxed{T_1 = 0,}$ (SM prediction)
- $\boxed{T_0 + T_2 = 0,}$ (SM prediction with $m_\ell = 0$)

In presence of NP contributions, the above two equations will not be satisfied. Therefore, in SM the angular distribution will be

$$\frac{d^2\Gamma}{ds\, d\cos\theta} = \frac{b\sqrt{s}}{128\,\pi^3\, m_B^2\left(m_B^2 - m_D^2 + s\right)}\left(T_0 + T_2\,\cos^2\theta\right), \quad \text{(SM prediction)}$$

which exhibits complete symmetry under $\cos\theta \longleftrightarrow -\cos\theta \equiv t \longleftrightarrow u$ exchange.

We define three angular asymmetries that are sensitive to the three distinct parts of the angular distribution.

$$A_0 \equiv A_0(s) = \frac{-\frac{1}{6}\left(\int_{-1}^{-\frac{1}{2}} -7\int_{-\frac{1}{2}}^{\frac{1}{2}} + \int_{\frac{1}{2}}^{1}\right)\frac{d^2\Gamma}{ds\, d\cos\theta}\, d\cos\theta}{d\Gamma/ds} = \frac{3}{2}\left(\frac{T_0}{3T_0+T_2}\right),$$

$$A_1 \equiv A_1(s) = \frac{-\left(\int_{-1}^{0} - \int_{0}^{1}\right)\frac{d^2\Gamma}{ds\, d\cos\theta}\, d\cos\theta}{d\Gamma/ds} = \frac{3}{2}\left(\frac{T_1}{3T_0+T_2}\right),$$

$$A_2 \equiv A_2(s) = \frac{2\left(\int_{-1}^{-\frac{1}{2}} - \int_{-\frac{1}{2}}^{\frac{1}{2}} + \int_{\frac{1}{2}}^{1}\right)\frac{d^2\Gamma}{ds\, d\cos\theta}\, d\cos\theta}{d\Gamma/ds} = \frac{3}{2}\left(\frac{T_2}{3T_0+T_2}\right).$$

Forward-backward asymmetry:

$$A_{FB} = -A_1 \propto T_1.$$

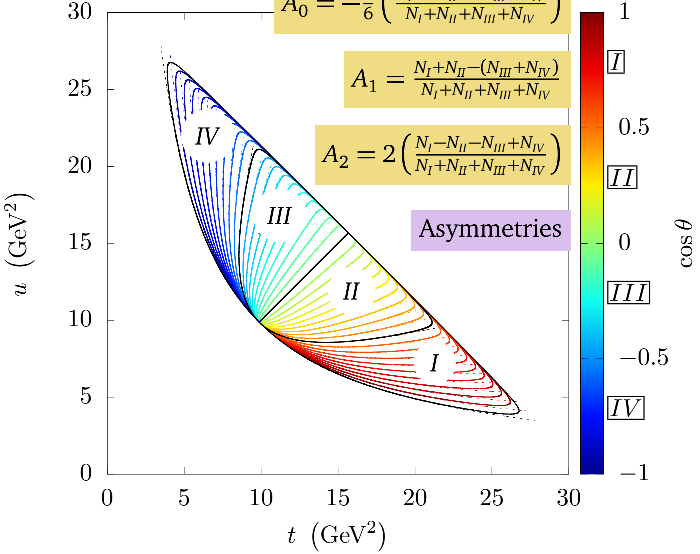
The three asymmetries are easily measurable using the Dalitz plot for $B \to D\ell^+\ell^-$ decays.

$$A_0 = -\frac{1}{6}\left(\frac{N_I - 7N_{II} - 7N_{III} + N_{IV}}{N_I + N_{II} + N_{III} + N_{IV}}\right)$$

$$A_1 = \frac{N_I + N_{II} - (N_{III} + N_{IV})}{N_I + N_{II} + N_{III} + N_{IV}}$$

$$A_2 = 2\left(\frac{N_I - N_{II} - N_{III} + N_{IV}}{N_I + N_{II} + N_{III} + N_{IV}}\right)$$

Asymmetries



We can define another asymmetry which probes the symmetry of the event distribution in the Dalitz plot.

We can define the following *binned asymmetry* A_{bin} to probe the symmetry of distribution of events in the Dalitz plot,

$$A_{\text{bin}} = \sum_{c\theta_m} \left| \frac{N(c\theta_m) - N(-c\theta_m)}{N(c\theta_m) + N(-c\theta_m)} \right|,$$

where we have divided the $B \to D\ell^+\ell^-$ Dalitz plot into an even number of segments after binning $\cos\theta$, and $N(c\theta_m)$ denotes the number of events in the segment centered around some $c\theta_m = \cos\theta_m$ with a bin width $2\Delta c\theta$.

In SM, we have

$$A_{\text{bin}} = 0.$$

New Physics in $B \to D\ell^+\ell^-$ decays leaves three simple model-independent signatures.

Signatures of NP:

- Non-zero forward-backward asymmetry: $A_1 \neq 0$, or $N_I \neq N_{IV}$ and $N_{II} \neq N_{III}$
- $A_0 + A_2 \neq 0$, and $A_0 \neq 3/4$, $A_2 \neq -3/4$, which is equivalent to $\dfrac{N_{II}}{N_I} \neq \dfrac{11}{5} \neq \dfrac{N_{III}}{N_{IV}}$.
- The distribution of events inside the Dalitz plot is not symmetric under $t \longleftrightarrow u$ exchange: $A_{\text{bin}} \neq 0$.

The model independent signatures of NP do not depend upon the kind of B or D meson we consider.

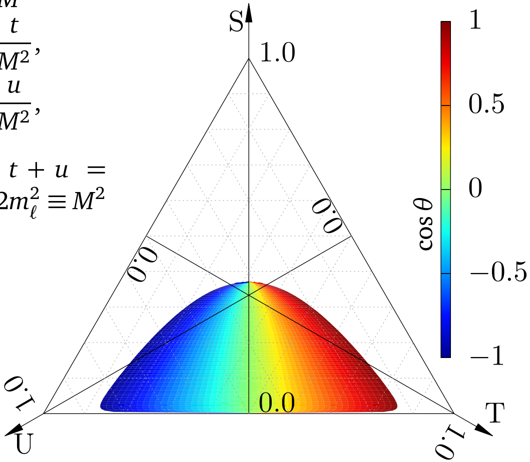
The model independent signatures of NP are the same for $B^0 \to D^0 \ell^+ \ell^-$, $B^0 \to \bar{D}^0 \ell^+ \ell^-$, $B^+ \to D^+ \ell^+ \ell^-$ as well as their CP conjugate modes, i.e. $\bar{B}^0 \to \bar{D}^0 \ell^+ \ell^-$, $\bar{B}^0 \to D^0 \ell^+ \ell^-$ and $B^- \to D^- \ell^+ \ell^-$.

These signatures are not affected by $B^0 - \bar{B}^0$ or $D^0 - \bar{D}^0$ mixings.

One can also look at a different kind of Dalitz plot, the ternary plot, to investigate the same signatures of NP.

$$S = \frac{s}{M^2},$$
$$T = \frac{t}{M^2},$$
$$U = \frac{u}{M^2},$$

where $s + t + u = m_B^2 + m_D^2 + 2m_\ell^2 \equiv M^2$ (say).



Conclusion

Any NP contribution in $B \to D\ell^+\ell^-$ decays, irrespective of the particulars of NP model, will leave three characteristic signatures in the corresponding Dalitz plot which can be quantified by easily observable angular asymmetries A_0, A_1 and A_2 and the binned asymmetry A_{bin}.

Thank you