Minimal extension of tribimaximal mixing and generalized $Z_2 \times Z_2$ symmetries

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The measurement of neutrino masses and mixings during the past decade has provided the first evidence for physics beyond the standard model (SM).

The neutrino mass matrix has nine parameters,

- three masses, three mixing angles and three CP violating phases if the neutrinos are Majorana particles

- The neutrino mass matrix is diagonalized by

\[
m_\nu = U \cdot \text{Diag}(m_1, m_2 e^{i\alpha}, m_3 e^{i\beta}) \cdot U^T
\]

\[
U = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\]
Introduction

- Experimental finding in this area has been the announcements by the T2K, and Double CHOOZ \(^1\), and recently by the Daya Bay and RENO \(^2\) experiments that one of the hitherto unknown neutrino mixing angles, namely \(\theta_{13}\), is not only non-zero but “large”.

- Large \(\theta_{13}\) have also been suggested by a global analysis of the existing oscillation data. \(^3\)

- The Tri-bimaximal mixing leads to the predictions on the three mixing angles.

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\(^1\) H. De. Kerrect [Double CHOOZ Collaboration], talk at LowNu conference in Korea (2011)


Tribimaximal Mixing

- \( \sin^2 \theta_{13} = 0 \),
- \( \sin^2 \theta_{23} = \pi/4 \) and
- \( \sin^2 \theta_{12} = 1/3 \)

For non zero \( \theta_{13} \)

- Perturbations to TBM pattern affecting mainly \( \theta_{13} \)
- Alternative flavour symmetries which imply non-zero \( \theta_{13} \)

The minimal scheme would be the one in which \( \theta_{13} \) is non zero but \( \theta_{23} \) and \( \theta_{12} \) remain close to their predictions in the TBM scheme.

The TBM pattern has the presence of specific \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) symmetry in the neutrino mass matrix \( M_{\nu_f} \) in the flavor basis. This symmetry is defined in general by the operators \( S_i \), \( i = 1, 2, 3 \):

\[
(S_i)_{jk} = \delta_{jk} - 2U_{ji}U^*_{ki} \tag{1}
\]

where \( U \) is the matrix diagonalizing \( M_{\nu_f} \). Each \( S_i \) defines a \( \mathbb{Z}_2 \) group.
Generalized $Z_2 \times Z_2$ symmetry

- $\mathcal{M}_{\nu_f}$ leads to the TBM \(^4\) if

$$S^T_{2,3} \mathcal{M}_{\nu_f} S_{2,3} = \mathcal{M}_{\nu_f}$$

where $S_{2,3}$ are the elements of the $Z_2 \times Z_2$ symmetry

$$S_2 = \frac{1}{3} \begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix} \quad \text{and} \quad S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$ 

$$S_2 S_3 = -S_1$$

- $S_2$ and $(S_3)$ are determined by the second (third) column of the TBM mixing matrix

- $S_3$ corresponds to well known $\mu-\tau$ symmetry

- $\theta_{13} = 0$, $\theta_{23} = \frac{\pi}{4}$

A desirable replacement of the $\mu$-$\tau$ symmetry would be which allows $\theta_{23}$ to be maximal but $\theta_{13}$ is not zero.

One such symmetry is obtained by combining $\mu$-$\tau$ symmetry with CP transformation:

$$S_3^T \mathcal{M}_{\nu f} S_3 = \mathcal{M}_{\nu f}^* .$$

This predicts:

$$\sin^2 \theta_{23} = \frac{1}{2},$$

$$\sin \theta_{13} \cos \delta = 0.$$

Either $\theta_{13} = 0$ or Dirac CP violation is maximal.

$\theta_{12}$ is unconstrained.

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Generalized $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry

We combine the generalized $\mu$-$\tau$ symmetry with the “magic symmetry” corresponding to invariance under $S_2$ and define a generalized $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry

Case I:

$$S_{1,3}^T M_{\nu_f} S_{1,3} = M_{\nu_f}^*.$$  

which implies

$$S_{2}^T M_{\nu_f} S_{2} = M_{\nu_f}.$$  

which fixes the second column of $V_{PMNS}$ matrix to be

$$1/\sqrt{3}(1, 1, 1)^T$$

$$|\sin \theta_{12} \cos \theta_{13}| = \frac{1}{\sqrt{3}} \implies \sin^2 \theta_{12} = \frac{1}{3} (1 + \tan^2 \theta_{13})$$

This puts a lower bound on $\theta_{12}$ i.e $\sin^2 \theta_{12} \geq 1/3$
Generalized $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry

\[
\mathcal{M}_{\nu f} = \begin{pmatrix}
  y + z - x & x + ix' & x - ix' \\
  x + ix' & y - ix' & z \\
  x - ix' & z & y + ix'
\end{pmatrix}
\]

- Re($\mathcal{M}_{\nu f}$) is in the TBM form while Im($\mathcal{M}_{\nu f}$) follows the condition

\[
S_{1,3}^T \text{ Im}(\mathcal{M}_{\nu f}) S_{1,3} = -\text{Im}(\mathcal{M}_{\nu f})
\]

- A special case where $\mathcal{M}_{\nu f}$ has 3 parameters\(^6\) can be obtained when

\[
x' = -\frac{1}{\sqrt{3}}(z - x).
\]

which results in smallest neutrino mass to be function of $\sin^2 \theta_{13}$ and $\Delta m^2_{\text{atm}}$

Case II: The second possibility is

\[ S_{2,3}^T M_{\nu f} S_{2,3} = M_{\nu f}^* , \]

which leads to

\[ S_1^T M_{\nu f} S_1 = M_{\nu f} . \]

This fixes the first column of \( V_{\text{PMNS}} \) to be \( 1/\sqrt{6}(2, -1, -1)^T \)

Which implies

\[ | \cos \theta_{12} \cos \theta_{13} | = \sqrt{\frac{2}{3}} \implies \sin^2 \theta_{12} = \frac{1}{3} \left( 1 - 2 \tan^2 \theta_{13} \right) . \]

An upper bound on the solar angle \( \sin^2 \theta_{12} \leq 1/3 \)
Generalized $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry

$$M_{\nu f} = \begin{pmatrix} y + z - x & x + ix' & x - ix' \\ x + ix' & y + 2ix' & z \\ x - ix' & z & y - 2ix' \end{pmatrix}$$

- **Case III:** $S_{1,2}^T M_{\nu f} S_{1,2} = M_{\nu f}^*$ and this results into the $\mu$-$\tau$ symmetric $M_{\nu f}$ which leads to $\theta_{13} = 0$
- Both cases I and II predict deviation from $\theta_{12}$ but in opposite direction
- Trivial Majorana phases and no restriction on masses of neutrinos
- Information on the five parameters, thus remaining four are unrestricted.
We extend the $A_4$ model for TBM by He et al.\cite{He:2006}.

- The matter and Higgs field content of the model with their assignments under the SM gauge group $G_{\text{SM}} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$ and $A_4$:

<table>
<thead>
<tr>
<th></th>
<th>$l_L$</th>
<th>$e_R$</th>
<th>$\mu_R$</th>
<th>$\tau_R$</th>
<th>$\nu_R$</th>
<th>$\Phi$</th>
<th>$\phi$</th>
<th>$\chi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{\text{SM}}$</td>
<td>$(1, 2, -1)$</td>
<td>$(1, 1, -2)$</td>
<td>$(1, 1, -2)$</td>
<td>$(1, 1, -2)$</td>
<td>$(1, 1, 0)$</td>
<td>$(1, 2, -1)$</td>
<td>$(1, 2, -1)$</td>
<td>$(1, 1, 0)$</td>
</tr>
<tr>
<td>$A_4$</td>
<td>3</td>
<td>1</td>
<td>$1'$</td>
<td>$1''$</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

- Yukawa interactions of the model:

$$-\mathcal{L}_Y = y_e (\bar{l}_L \tilde{\Phi})_1 e_R + y_\mu (\bar{l}_L \tilde{\Phi})_1'' \mu_R + y_\tau (\bar{l}_L \tilde{\Phi})_1' \tau_R$$

$$+ y_D (\bar{l}_L \nu_R) \phi + \frac{1}{2} M \tilde{\nu}_R \nu_R^c + \frac{1}{2} B' (\tilde{\nu}_R \nu_R^c)_3 \chi + \text{h.c.}$$

$\tilde{\Phi} \equiv i \tau_2 \Phi^*$ We assume that all the couplings and all the vevs of scalar fields are real.

\cite{He:2006} X. -G. He, Y. -Y. Keum, R. R. Volkas, JHEP 0604, 039 (2006)
After the electroweak symmetry breaking governed by the vev of $\Phi$ and $\phi$ and if it is assumed that $\langle \Phi \rangle = \nu(1, 1, 1)^T$ then

$$M_l = \sqrt{3} \nu \ U(\omega) \ \text{Diag.}(y_e, y_\mu, y_\tau),$$

where

$$U(\omega) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$$

and $\omega = e^{i2\pi/3}$

$$M_D = y_D \nu_\phi I$$

where $\nu_\phi = \langle \phi \rangle$

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Further, assuming that the field $\chi$ develops vev in the direction $\langle \chi \rangle = \nu_{\chi}(1, 0, 0)^T$, the heavy neutrino mass matrix can be written as

$$M_R = \begin{pmatrix} A & 0 & 0 \\ 0 & A & B \\ 0 & B & A \end{pmatrix}$$

$B = B'\nu_{\chi}$

After the seesaw

$$M^I_{\nu} = -M_D M_R^{-1} M_D^T = \begin{pmatrix} (a^2 - b^2) & 0 & 0 \\ a & a & b \\ 0 & b & a \end{pmatrix}$$

$$a = -\frac{y_D^2 \nu^2}{A^2 - B^2} A \text{ and } b = \frac{y_D^2 \nu^2}{A^2 - B^2} B$$
We extend the model to get the desired neutrino mass matrix in two different ways

- Adding Higgs triplet through type-II seesaw
- Adding flavon field through type-I seesaw
- By adding three copies of SU(2) triplet fields $\Delta$ which also form a triplet of $A_4$ the Lagrangian has an additional term

$$\mathcal{L}_{\Delta} = y_L (\bar{l}_L l_L^c)_{3} \Delta + h.c.$$  

$$\langle \Delta \rangle = \nu_{\Delta} (0, -1, 1)^T$$  

$$\mathcal{M}_{\nu}^{II} = \begin{pmatrix}
0 & c & -c \\
c & 0 & 0 \\
-c & 0 & 0
\end{pmatrix}$$

$$\mathcal{M}_{\nu} = \mathcal{M}_{\nu}^{I} + \mathcal{M}_{\nu}^{II}$$
The neutrino mass matrix in flavor basis

\[ M_{\nu_f} = U(\omega)^T M_{\nu} U(\omega) \]

when compared with general neutrino mass matrix derived earlier has an additional restriction on the parameters. This additional constraint restricts the neutrino masses. This mass matrix has 3 parameters. Thus there is an additional restriction on neutrino masses.

In exact TBM limit, this restriction results into following sum rule

\[ \frac{2}{m_2} + \frac{1}{m_3} = \frac{1}{m_1} \]
Figure: Correlations of $\theta_{13}$ with mass dependent observables

- All the mass dependent observables slightly vary with the reactor angle

- Another extension of the model is by adding $A_4$ triplet flavon field $\chi'$ the Lagrangian has an additional term.

$$-\mathcal{L}^{\chi'}_Y = \frac{1}{2} y_R (\overline{\nu}_R \nu^c_R)_3 \chi' + \text{h.c.}$$
Assuming that $\chi'$ takes a vev $\langle \chi' \rangle = \nu_{\chi'}(0, -1, 1)^T$

$$M_R = \begin{pmatrix} A & C & -C \\ C & A & B \\ -C & B & A \end{pmatrix}$$

where $C = y_R \nu_{\chi'}$

$$M_L = \begin{pmatrix} \frac{(a^2 - b^2 + c^2)}{a} & c & -c \\ a & \frac{c}{a} & a & b \\ -c & b & a \end{pmatrix}$$

$$a = \frac{(C^2 - A^2)\nu_\phi^2 y_D^2}{(A+B)(A^2 - AB - 2C^2)}$$
$$b = \frac{(C^2 + AB)\nu_\phi^2 y_D^2}{(A+B)(A^2 - AB - 2C^2)}$$
and
$$c = \frac{C\nu_\phi^2 y_D^2}{(A^2 - AB - 2C^2)}$$
In the limit of exact TBM For NH

\[ \Sigma m_i = 0.064\text{eV}, 0.066\text{eV} \quad m_\beta = 0.006\text{eV}, 0.008\text{eV} \quad m_{\beta\beta} = 0.0052\text{eV}, 0.0054\text{eV} \]

For IH \[ \Sigma m_i = 0.122\text{eV} \quad m_\beta = 0.052\text{eV} \quad m_{\beta\beta} = 0.0156\text{eV} \]

The phenomenology of neutrino masses does not change significantly from the previous case.
The latest neutrino oscillation data suggest non zero $\theta_{13}$, that suggest one to look for perturbations to the TBM pattern or search for alternate flavor symmetry.

A class of symmetry can be obtained by combining $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry with CP (generalized $\mathbb{Z}_2 \times \mathbb{Z}_2$)

Imposition of such symmetry lead to

I) $\sin^2 \theta_{23} = \frac{1}{2}$, $\sin^2 \theta_{12} = \frac{1}{3}(1 + \tan^2 \theta_{13})$

II) $\sin^2 \theta_{23} = \frac{1}{2}$, $\sin^2 \theta_{12} = \frac{1}{3}(1 - 2 \tan^2 \theta_{13})$, $\delta = \pi/2$

Case II can be obtained by extending the $A_4$ model either through SU(2) triplet or flavon field. The model leads to prediction for absolute neutrino mass as a function of reactor angle which can be probed in future non oscillation experiments.
THANK YOU