Minimal extension of tribimaximal mixing and generalized $Z_2 \times Z_2$ symmetries

S. Gupta

A. S. Joshipura and K. M. Patel

Introduction

- The measurement of neutrino masses and mixings during the past decade has provided the first evidence for physics beyond the standard model (SM).

The neutrino mass matrix has nine parameters,

- three masses, three mixing angles and three CP violating phases if the neutrinos are Majorana particles

- The neutrino mass matrix is diagonalized by

$$\mathbf{m}_\nu = \mathbf{U.Diag(m_1, m_2 e^{i\alpha}, m_3 e^{i\beta}).U^T}$$

$$\mathbf{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Introduction

- Experimental finding in this area has been the announcements by the T2K, and Double CHOOZ [1], and recently by the Daya Bay and RENO [2] experiments that one of the hitherto unknown neutrino mixing angles, namely θ_{13}, is not only non-zero but "large".

- Large θ_{13} have also been suggested by a global analysis of the existing oscillation data. [3]

- The Tri-bimaximal mixing leads to the predictions on the three mixing angles

[1] H. De. Kerrect [Double CHOOZ Collaboration], talk at LowNu conference in Korea (2011)

[2] F. P. An *et al.*, Phys. Rev. Lett. **108**, 171803 (2012); J. K. Ahn *et al.* [RENO Collaboration], Phys. Rev. Lett. **108**, 191802 (2012)

[3] G. L. Fogli *et al.*, Phys. Rev. D **84**, 053007; T. Schwetz, M. Tortola and J. W. F. Valle, New J. Phys. **13**, 109401 (2011)

Tribimaximal Mixing

- $\sin\theta_{13} = 0$,
  $\sin^2\theta_{23} = \pi/4$ and
  $\sin^2\theta_{12} = 1/3$

  For non zero θ_{13}

- Perturbations to TBM pattern affecting mainly θ_{13}
- Alternative flavour symmetries which imply non-zero θ_{13}

The minimal scheme would be the one in which θ_{13} is non zero but θ_{23} and θ_{12} remain close to their predictions in the TBM scheme

The TBM pattern has the presence of specific $Z_2 \times Z_2$ symmetry in the neutrino mass matrix $M_{\nu f}$ in the flavor basis This symmetry is defined in general by the operators S_i , $i = 1, 2, 3$:

$$(S_i)_{jk} = \delta_{jk} - 2U_{ji}U^*_{ki} \ , \quad (1)$$

where U is the matrix diagonalizing $\mathcal{M}_{\nu f}$. Each S_i defines a Z_2 group

Generalized $Z_2 \times Z_2$ symmetry

- $\mathcal{M}_{\nu f}$ leads to the TBM [4] if

$$S_{2,3}^T \mathcal{M}_{\nu f} S_{2,3} = \mathcal{M}_{\nu f}$$

where $S_{2,3}$ are the elements of the $Z_2 \times Z_2$ symmetry

$$S_2 = \frac{1}{3}\begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix} \quad \text{and} \quad S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

$$S_2 S_3 = -S_1$$

- S_2 and (S_3) are determined by the second (third) column of the TBM mixing matrix
- S_3 corresponds to well known μ-τ symmetry
- $\theta_{13} = 0, \theta_{23} = \frac{\pi}{4}$

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[4] C. S. Lam, Phys. Rev. Lett. **101** 121602 (2008)

Generalized $Z_2 \times Z_2$ symmetry

- A desirable replacement of the μ-τ symmetry would be which allows θ_{23} is maximal but θ_{13} is non zero
- One such symmetry is obtained by combining μ-τ symmetry with CP transformation [5]

$$S_3^T \mathcal{M}_{\nu f} S_3 = \mathcal{M}_{\nu f}^* \ .$$

- This predicts

$$\begin{aligned} \sin^2\theta_{23} &= \frac{1}{2}, \\ \sin\theta_{13}\cos\delta &= 0. \end{aligned}$$

- Either $\theta_{13} = 0$ or Dirac CP violation is maximal. θ_{12} is unconstrained

[5] W. Grimus, L. Lavoura, Phys. Lett. **B579**, 113-122 (2004)

Generalized $Z_2 \times Z_2$ symmetry

- We combine the generalized μ-τ symmetry with the "magic symmetry" corresponding to invariance under S_2 and define a generalized $Z_2 \times Z_2$ symmetry
- **Case I:**

$$S_{1,3}^T \mathcal{M}_{\nu f} S_{1,3} = \mathcal{M}_{\nu f}^*.$$

which implies

$$S_2^T \mathcal{M}_{\nu f} S_2 = \mathcal{M}_{\nu f}.$$

which fixes the second column of V_{PMNS} matrix to be $1/\sqrt{3}(1,1,1)^T$

$$|\sin\theta_{12}\cos\theta_{13}| = \frac{1}{\sqrt{3}} \implies \sin^2\theta_{12} = \frac{1}{3}(1+\tan^2\theta_{13})$$

- This puts a lower bound on θ_{12} *i.e* $\sin^2\theta_{12} \geq 1/3$

Generalized $Z_2 \times Z_2$ symmetry

$$\mathcal{M}_{\nu f} = \begin{pmatrix} y+z-x & x+ix' & x-ix' \\ x+ix' & y-ix' & z \\ x-ix' & z & y+ix' \end{pmatrix}$$

- $\mathrm{Re}(\mathcal{M}_{\nu f})$ is in the TBM form while $\mathrm{Im}(\mathcal{M}_{\nu f})$ follows the condition

$$S_{1,3}^T \, \mathrm{Im}(\mathcal{M}_{\nu f}) \, S_{1,3} = -\mathrm{Im}(\mathcal{M}_{\nu f})$$

- A special case where $\mathcal{M}_{\nu f}$ has 3 parameters [6] can be obtained when

$$x' = -\frac{1}{\sqrt{3}}(z-x).$$

which results in smallest neutrino mass to be function of $\sin^2\theta_{13}$ and Δm^2_{atm}

[6] W. Grimus, L. Lavoura, Phys. Lett. **B671**, 456-461 (2009)

Generalized $Z_2 \times Z_2$ symmetry

- **Case II:** The second possibility is

$$S_{2,3}^T \mathcal{M}_{\nu f} S_{2,3} = \mathcal{M}_{\nu f}^* \ ,$$

  which leads to

$$S_1^T \mathcal{M}_{\nu f} S_1 = \mathcal{M}_{\nu f}.$$

  This fixes the first column of V_{PMNS} to be $1/\sqrt{6}(2, -1, -1)^T$
- Which implies

$$|\cos\theta_{12}\cos\theta_{13}| = \sqrt{\frac{2}{3}} \implies \sin^2\theta_{12} = \frac{1}{3}(1 - 2\tan^2\theta_{13}).$$

- An upper bound on the solar angle $\sin^2\theta_{12} \leq 1/3$

Generalized $Z_2 \times Z_2$ symmetry

$$\mathcal{M}_{\nu f} = \begin{pmatrix} y+z-x & x+ix' & x-ix' \\ x+ix' & y+2ix' & z \\ x-ix' & z & y-2ix' \end{pmatrix}$$

- **Case III:** $S_{1,2}^T \mathcal{M}_{\nu f} S_{1,2} = \mathcal{M}_{\nu f}^*$ and this results into the μ-τ symmetric $\mathcal{M}_{\nu f}$ which leads to $\theta_{13} = 0$
- Both cases I and II predict deviation from θ_{12} but in opposite direction
- Trivial Majorana phases and no restriction on masses of neutrinos
- Information on the five parameters, thus remaining four are unrestricted.

We extend the $\mathbf{A_4}$ model for TBM by He *et al* [7]

- The matter and Higgs field content of the model with their assignments under the SM gauge group $\mathbf{G_{SM} \equiv SU(3)c \times SU(2)_L \times U(1)_Y}$ and $\mathbf{A_4}$

	l_L	e_R	μ_R	τ_R	ν_R	Φ	ϕ	χ
G_{SM}	(1, 2, −1)	(1, 1, −2)	(1, 1, −2)	(1, 1, −2)	(1, 1, 0)	(1, 2, −1)	(1, 2, −1)	(1, 1, 0)
A_4	3	1	1′	1′′	3	3	1	3

- Yukawa interactions of the model

$$
\begin{aligned}
-\mathcal{L}_Y &= y_e(\bar{l}_L\tilde{\Phi})_1 e_R + y_\mu(\bar{l}_L\tilde{\Phi})_{1''}\mu_R + y_\tau(\bar{l}_L\tilde{\Phi})_{1'}\tau_R \\
&+ y_D(\bar{l}_L\nu_R)\phi + \frac{1}{2}M\overline{\nu}_R\nu_R^c + \frac{1}{2}B'(\overline{\nu}_R\nu_R^c)_3\chi + \text{h.c.}
\end{aligned}
$$

$\tilde{\Phi} \equiv i\tau_2\Phi^*$ We assume that all the couplings and all the vevs of scalar fields are real

[7] X. -G. He, Y. -Y. Keum, R. R. Volkas, JHEP **0604**, 039 (2006)

- After the electroweak symmetry breaking governed by the vev of $\mathbf{\Phi}$ and ϕ and if it is assumed that $\langle \mathbf{\Phi} \rangle = \upsilon(1,\ 1,\ 1)^{\mathsf{T}}$ [8] then

$$\mathsf{M_l} = \sqrt{3}\upsilon\ \mathsf{U}(\omega)\ \mathrm{Diag.}(\mathsf{y_e},\ \mathsf{y_\mu},\ \mathsf{y_\tau})\ ,$$

where

$$\mathsf{U}(\omega) = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$$

and $\omega = \mathsf{e}^{\mathsf{i}2\pi/3}$

- $$\mathsf{M_D} = \mathsf{y_D}\upsilon_\phi \mathsf{I}$$

where $\upsilon_\phi = \langle \phi \rangle$

[8] X. -G. He, Y. -Y. Keum, R. R. Volkas, JHEP **0604**, 039 (2006)

Model & Phenomenology

- Further, assuming that the field χ develops vev in the direction $\langle\chi\rangle = v_\chi(1,\ 0,\ 0)^T$, the heavy neutrino mass matrix can be written as

$$M_R = \begin{pmatrix} A & 0 & 0 \\ 0 & A & B \\ 0 & B & A \end{pmatrix}$$

$$B = B' v_\chi$$

- After the seesaw

$$\mathcal{M}_\nu^I = -M_D M_R^{-1} M_D^T = \begin{pmatrix} \frac{(a^2-b^2)}{a} & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{pmatrix}$$

$$a = -\frac{y_D^2 v_\phi^2}{A^2 - B^2} A \text{ and } b = \frac{y_D^2 v_\phi^2}{A^2 - B^2} B$$

Model & Phenomenology

We extend the model to get the desired neutrino mass matrix in two different ways

- Adding Higgs triplet through type-II seesaw
- Adding flavon field through type-I seesaw
- By adding three copies of SU(2) triplet fields $\mathbf{\Delta}$ which also form a triplet of $\mathbf{A_4}$ the Lagrangian has an additional term

$$-\mathcal{L}_Y^{\Delta} = y_L(\bar{l}_L l_L^c)_3 \Delta + h.c.$$

$$\langle \Delta \rangle = v_\Delta (0, -1, 1)^T$$

$$\mathcal{M}_\nu^{II} = \begin{pmatrix} 0 & c & -c \\ c & 0 & 0 \\ -c & 0 & 0 \end{pmatrix}$$

$$\mathcal{M}_\nu = \mathcal{M}_\nu^{I} + \mathcal{M}_\nu^{II}$$

$$\mathcal{M}_\nu = \begin{pmatrix} \frac{(a^2-b^2)}{a} & c & -c \\ c & a & b \\ -c & b & a \end{pmatrix} \tag{2}$$

- The neutrino mass matrix in flavor basis

$$\mathcal{M}_{\nu f} = U(\omega)^T \mathcal{M}_\nu U(\omega)$$

  when compared with general neutrino mass matrix derived earlier has an additional restriction on the parameters. This additional constraint restricts the neutrino masses. This mass matrix has 3 parameters. Thus there is an additional restriction on neutrino masses.

- In exact TBM limit, this restriction results into following sum rule

$$\frac{2}{m_2} + \frac{1}{m_3} = \frac{1}{m_1}$$

Model & Phenomenology

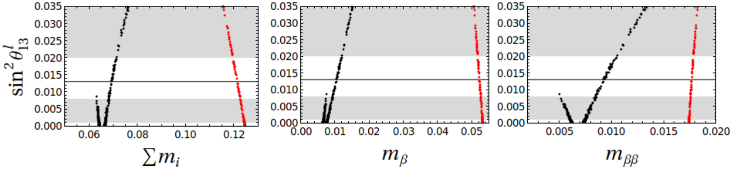


Figure: Correlations of θ_{13} with mass dependent observables

- All the mass dependent observables slightly vary with the reactor angle
- Another extension of the model is by adding $\mathbf{A_4}$ triplet flavon field χ' the Lagrangian has an additional term.

$$-\mathcal{L}_{\mathbf{Y}}^{\chi'} = \frac{1}{2} y_R (\overline{\nu}_R \nu_R^c)_3 \chi' + \mathbf{h.c.}$$

- Assuming that χ' takes a vev $\langle\chi'\rangle = v_{\chi'}(0,-1,1)^T$

$$M_R = \begin{pmatrix} A & C & -C \\ C & A & B \\ -C & B & A \end{pmatrix}$$

where $C = y_R v_{\chi'}$

- $$\mathcal{M}_\nu = \begin{pmatrix} \frac{(a^2-b^2+c^2)}{a} & c & -c \\ c & a & b \\ -c & b & a \end{pmatrix}$$

$$a = \frac{(C^2-A^2)v_\phi^2 y_D^2}{(A+B)(A^2-AB-2C^2)}, b = \frac{(C^2+AB)v_\phi^2 y_D^2}{(A+B)(A^2-AB-2C^2)} \text{ and}$$

$$c = \frac{C v_\phi^2 y_D^2}{(A^2-AB-2C^2)}$$

Model & Phenomenology

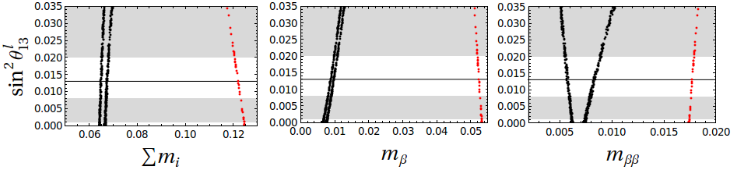
- 

Figure: Correlations between θ_{13} and neutrino mass dependent observables

In the limit of exact TBM For NH

$\Sigma m_i = 0.064\text{eV}, 0.066\text{eV}, \quad m_\beta = 0.006\text{eV}, 0.008\text{eV} \quad , m_{\beta\beta} = 0.0052\text{eV}, 0.0054\text{eV}$

For IH $\Sigma m_i = 0.122\text{eV}, \quad m_\beta = 0.052\text{eV} \quad , m_{\beta\beta} = 0.0156\text{eV}$

- The phenomenology of neutrino masses does not change significantly from the previous case

Conclusions

- The latest neutrino oscillation data suggest non zero θ_{13}, that suggest one to look for perturations to the TBM pattern or search for alternate flavor symmetry.
- A class of symmetry can be obtained by combining $\mathbf{Z_2} \times \mathbf{Z_2}$ symmetry with CP (generalized $\mathbf{Z_2} \times \mathbf{Z_2}$)
- Imposition of such symmetry lead to

$$\mathbf{I)}\ \sin^2\theta_{23} = \frac{1}{2}, \sin^2\theta_{12} = \frac{1}{3}(1 + \tan^2\theta_{13})$$

$$\mathbf{II)}\ \sin^2\theta_{23} = \frac{1}{2}, \sin^2\theta_{12} = \frac{1}{3}(1 - 2\tan^2\theta_{13}), \delta = \pi/2$$

- Case II can be obtained by extending the $\mathbf{A_4}$ model either through SU(2) triplet or flavon field. The model leads to prediction for absolute neutrino mass as a function of reactor angle which can be probed in future non oscillation experiments

THANK YOU